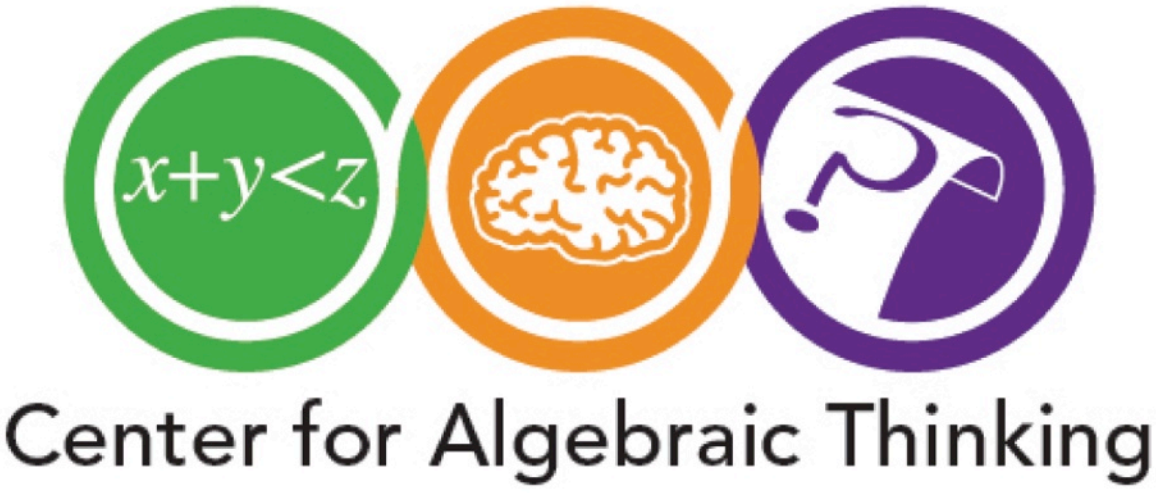
Teaching Resources

For iPad Apps Developed by the
Center for Algebraic Thinking



App Resources

A master list of materials developed by
the Center for Algebraic Thinking

Center for Algebraic Thinking

Assignment Types
- Screenshot Presentation Teaching Guide

Resources by App

Action Grapher
- Developing Game Strategies (Teaching Guide)
- Filling the Flask (Teaching Guide)
- The Flask Challenge (Teaching Guide)
- The Doodle Challenge (Teaching Guide)
- The Doodle Duel (Teaching Guide)
- Doodle Pictionary (Teaching Guide)
    - ✦ Doodle Pictionary: Shapes

Algebra Card Clutter
- Two-Player Card Clutter **NEW** (Teaching Guide) **NEW**
- Card Clutter Progress Monitor **NEW** (Teaching Guide) **NEW**
- Student-Created Card Clutter **NEW** (Teaching Guide) **NEW**

Cover Up Math
- Sticky Note Cover Up Math (Teaching Guide)
- Cover Up Math App Progress Monitor (Teaching Guide)
- Sticky Note Cover Up Challenge (Teaching Guide
- Comparison of Methods: Cover Up VS Algorithmic (Teaching Guide)
- The Mental Math Cover Up Challenge (Teaching Guide)

Diamond Factor
- Developing Game Strategies (Teaching Guide)
- Deriving the Diamond Method - Part 1 (Teaching Guide)
- Deriving the Diamond Method - Part 2 (Teaching Guide)

Function Mystery Machine
- Developing Game Strategies (Teaching Guide)
- Discovering the Mystery Function (Teaching Guide)

Linear Model
- Drag and Drop Linear Equations (Teaching Guide)
- Multiple Representations of Lines (Teaching Guide)

Lion Grapher
- Developing Game Strategies (Teaching Guide)

- Line-Building Challenge (Teaching Guide)

Math Flyer

- Function Investigation Series (Teaching Guide)
  - Linear
  - Quadratic (Part 1) [$y = ax^2 + bx + c$]
  - Quadratic (Part 2) [$y = a(x - h)^2 + k$]
  - Cosine
  - Sine
  - Absolute Value
- Graphing the Terms of Polynomials (Teaching Guide)

Point Plotter

- Developing Game Strategies (Teaching Guide)
- Moving from Points to Lines (Teaching Guide)
- Moving from Points to Lines: The Follow Up **COMPLETE** (Teaching Guide) **NEW**

Slope Slider

- Function Investigation - Linear (Teaching Guide)

Submariner Algebra

- Developing Game Strategies (Teaching Guide)
- Submariner Algebra Workspace **NEW** (Teaching Guide) **NEW**
- Submariner Algebra: The Paper Version **NEW** (Teaching Guide) **NEW**

Tortoise and the Hare Algebra

- Tortoise and the Hare Algebra Challenge Progress Monitor (Teaching Guide) **NEW**

Activity Progressions

Suggested sequences of activities to use with Center for Algebraic Thinking resources

The Cover Up Math Method

1) Sticky Note Cover Up Math
2) Cover Up Math App Progress Monitor
3) Sticky Note Cover Up Challenge (selecting the appropriate level)
4) Comparison of Methods: Cover Up VS Algorithmic
5) The Mental Math Cover Up Challenge

Linear Equations

1) Moving from Points to Lines
2) Moving from Points to Lines: The Follow Up
3) Drag and Drop Linear Equations
4) Multiple Representations of Linear Equations
5) Line Building Challenge

Rate of Change Applications

TEACHING GUIDE

Assignment Type: Screenshot Presentation

Overview:

Students demonstrate proficiency by taking screenshots of accomplishments and compiling those screenshots into a presentation to share with the instructor and/or the class.

Common Core State Standards:

The standards that this type of assignment addresses are specific to the objectives of each individual assignment.

Encyclopedia of Algebraic Thinking:

The relevant entries of the Encyclopedia of Algebraic Thinking are specific to the objectives of each individual assignment.

Description:

Rather than students turning in a worksheet as a record of their work, this assignment incorporates the tools available on the iPad to allow students to demonstrate knowledge in a more interactive way - they compile their own evidence of accomplishment and present those accomplishments in presentation form. The outline of the process is the following:

- The instructor provides the objectives. Students must be able to meet these objectives using an app on the iPad.
- For each objective that is met, the student takes a screenshot to demonstration completion.
- The student compiles those screenshots into a presentation (using the Keynote app, for example), adding text and other media to the presentation as needed.
- The student shows/emails the presentation to the instructor or presents it to the class.

Extensions:

In addition to demonstrating completion of objectives, students could add additional slides to discuss difficulties faced while meeting those objectives and how they overcame those difficulties.

Developing Game Strategies

For use with any of the apps developed by the Center for Algebraic Thinking

Name: Date: Period:

Name of app: _____

Summarize the challenge that you are faced with while playing this game (include a sketch of the game environment):

What is *your* strategy for succeeding at this task?

Talk with classmates about the strategies they use while playing this game. How are their strategies different than yours? Whose strategies are more effective, yours or theirs? Why?

Play the game again, and see if you do any better now that you have more strategies to use!

TEACHING GUIDE

Developing Game Strategies

Overview:

Students describe the app, outline their own strategies for succeeding at the task at hand, and share their strategies with their classmates.

Common Core State Standards:

- Standard for Mathematical Practice #3
- [Additional standards will apply that are specific to the app with which this activity is used]

Encyclopedia of Algebraic Thinking:

The relevant entries of the Encyclopedia of Algebraic Thinking are specific to the app with which this activity is used.

Description:

The purpose of this activity is to encourage students to pay particular attention to the way in which they approach a task. This activity also incorporates the social aspect of learning - the students experiment on their own and then share strategies with each other - which develops longer-lasting skills then they would if the teacher explained “the one best method” prior to the students embarking on the activity.

Extensions:

A follow-up assignment could be for students to monitor their success both before they discuss their strategies with classmates and after, and then reflect on the process.

Filling the Flask

For use with the iPad app *Action Grapher*

Center for Algebraic Thinking

Name: Date: Period:

Part 1: Explore

Navigate to the "Flask" tab of the iPad app *Action Grapher*. Read the instructions, and then see if you can select the height vs. time graph that corresponds to the given flask. Also try creating your own custom flasks. Once you can consistently select the correct graph for many different flasks, you can move on to Part 2 below.

Part 2: Explain

Now that you have had a chance to explore this section of the app, your next task is to explain the relationship between the side view of the flask and the graph of height vs. time. To begin, sketch out both the flask and the height vs. time graph of a non-custom flask (i.e., a flask that is automatically generated by the app):

Flask	**Height vs. Time Graph**

Now add a horizontal dotted line to your sketch of the flask to separate unique sections of the flask. Then add a *vertical* dotted line to each unique section of the height vs. time graph. Finally, number each section of your flask sketch and height vs. time graph such that Section 1 of your flask sketch corresponds to Section 1 of your height vs. time graph.

Use the table below to explain what is going on in each section that you labeled on the previous page:

Section	What is happening with the flask in this section?	What is happening with the height vs. time graph in this section?	Do the changes in the flask and in the graph make sense together? Explain.

TEACHING GUIDE

Filling the Flask

Center for Algebraic Thinking

Overview:

Students develop and solidify the concept of rate of change as it pertains to the change in the height of liquid in a student-drawn flask as a function of time (at a constant rate of pouring).

Common Core State Standards:

- F-IF.4
- F-IF.6

Encyclopedia of Algebraic Thinking:

- Analysis of Change: Interpreting Graphs
- Analysis of Change: Graphs as a Literal Picture

Description:

This activity will serve as a good introduction to the Flask section of the app Action Grapher. After students are given time to freely explore this section of the app in Part 1 of the activity, in Part 2 they must explain in detail the relationship between the two different representations - the side view of the flask and the graph of height vs. time. They are provided with enough prompts along the way - with particular attention directed to the unique sections of the height vs time graph - so that they can form a complete and consistent view of the connection between the two representations.

Extensions:

- The Flask Challenge

The Flask Challenge

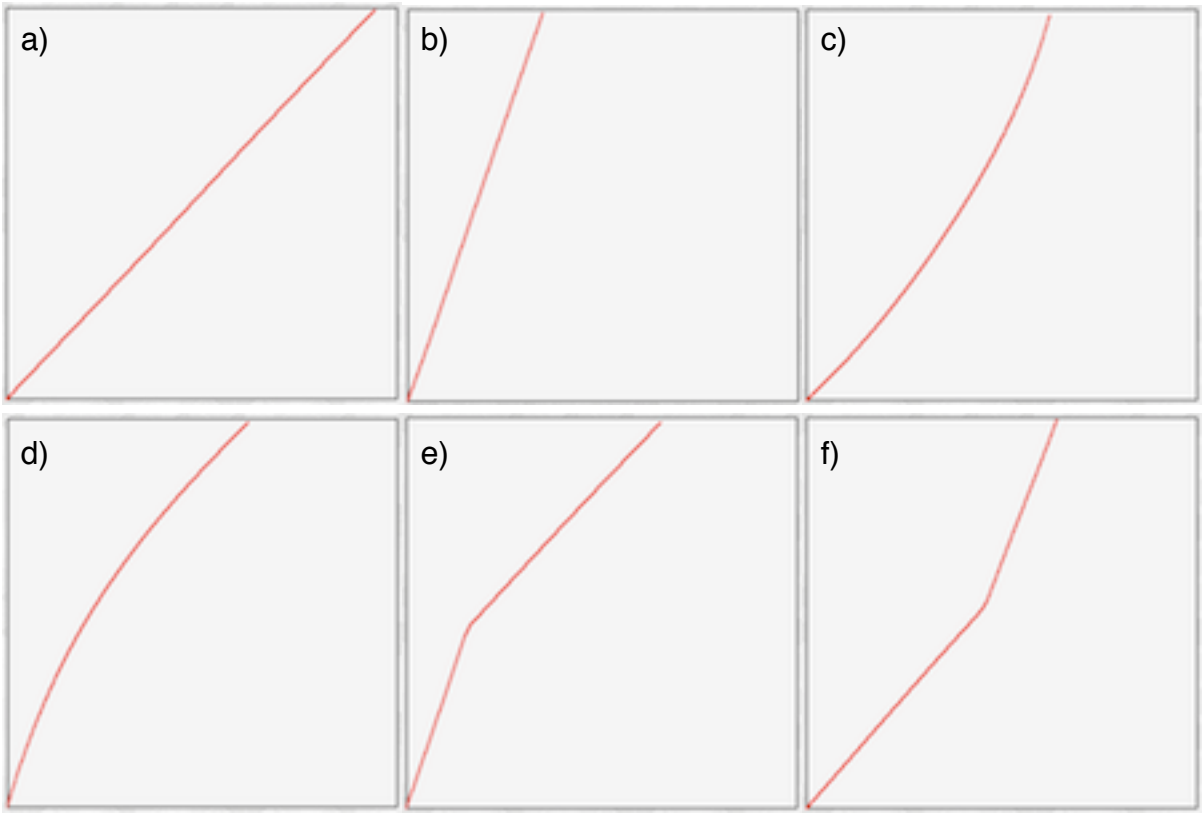
For use with the iPad app *Action Grapher*

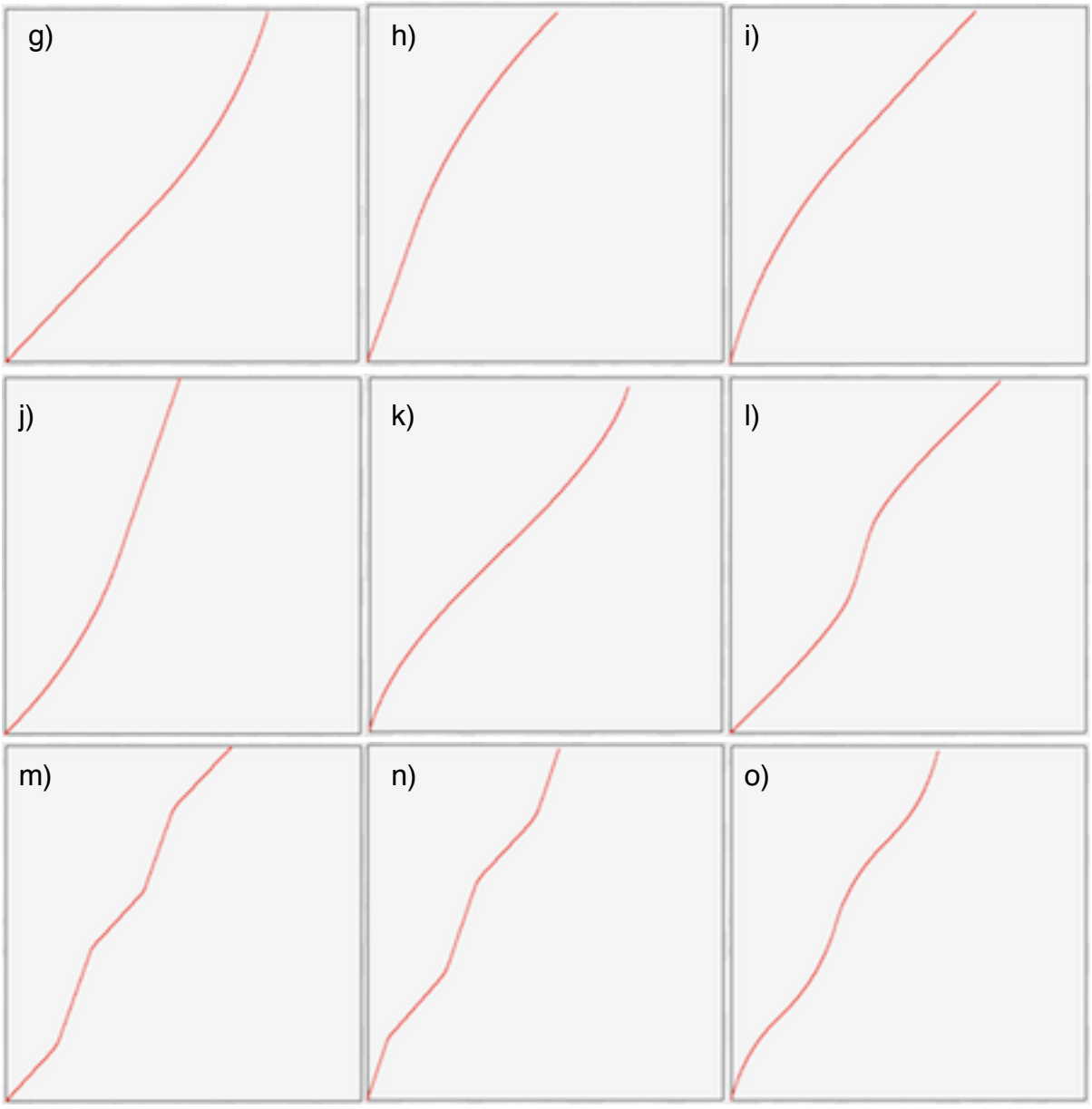
Name: Date: Period:

This is a Screenshot Presentation assignment. Here are your objectives:

Objective 1:

Navigate to the "Flask" tab of the iPad app *Action Grapher*. For each of the following, draw a custom flask that will produce a height vs. time graph that is the same as the graph shown:




g)
h)
i)
j)
k)
l)
m)
n)
o)

TEACHING GUIDE

The Flask Challenge

Overview:

Students draw liquid containers in the particular shape that will produce the given height vs. time graph when the container is filled at a constant rate of pouring.

Common Core State Standards:

- F-IF.4
- F-IF.6

Encyclopedia of Algebraic Thinking:

- Analysis of Change: Interpreting Graphs
- Analysis of Change: Graphs as a Literal Picture

Description:

In this activity students will get practice switching back and forth between two different representations of rate of change. Students are likely already familiar with one of those representations - the filling of a liquid container. They will be less familiar with the second representation - a graph of liquid height in the container as a function of time (when the container is filled at a constant rate of pouring). There is enough variation in the provided graphs of height vs. time that after completing all of them students will have solidified the connection between the two representations. The graphs are ordered by complexity - from the simplest to the most complex.

Extensions:

Two-Player Challenge: one students draws h vs. t graph, the other must draw the flask that produces that graph.

The Doodle Challenge

For use with the iPad app *Action Grapher*

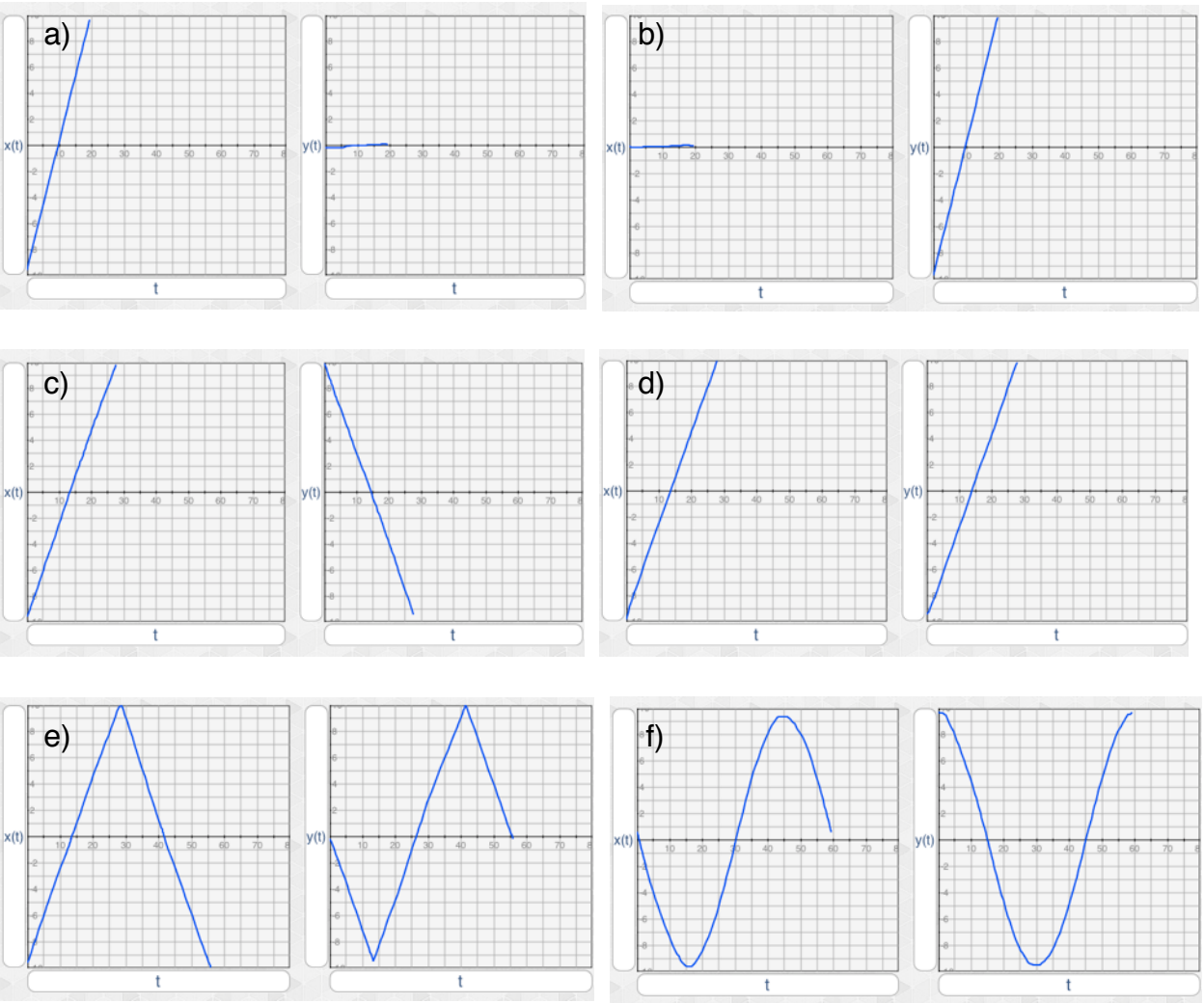
Center for Algebraic Thinking

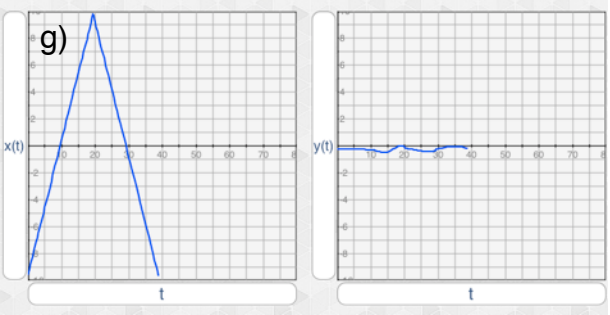
Name: Date: Period:

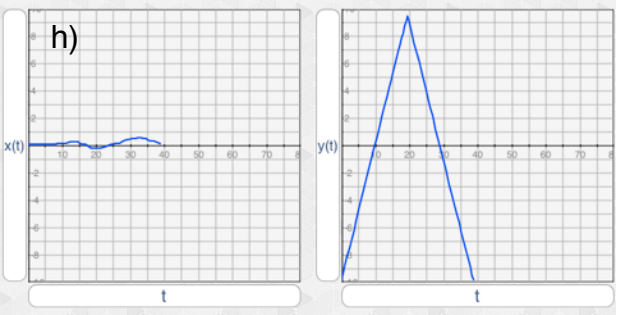
This is a Screenshot Presentation assignment. Here are your objectives:

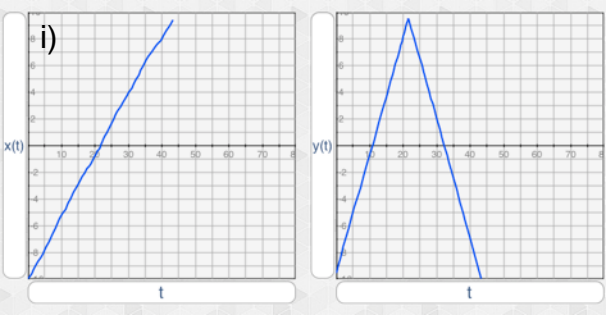
Objective 1:

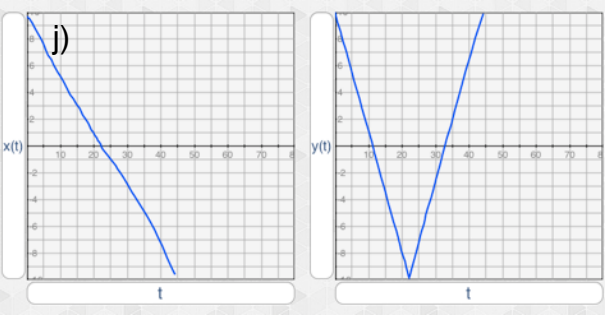
Navigate to the "Doodle" tab of the iPad app *Action Grapher*. From the Home page (not Game One nor Game Two), draw a doodle that produces the following x vs. arc length and y vs. arc length graphs:

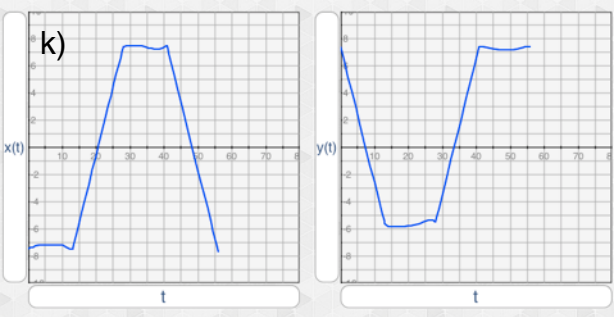


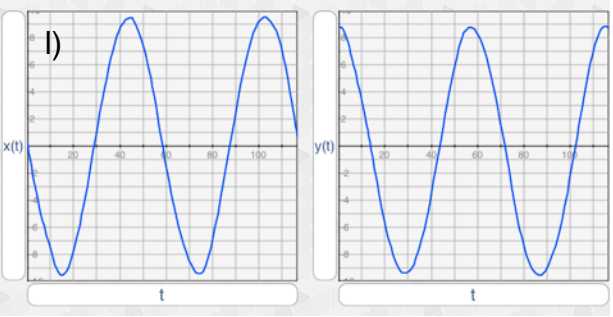

g)
x(t)
y(t)
t
t


h)
x(t)
y(t)
t
t


i)
x(t)
y(t)
t
t


j)
x(t)
y(t)
t
t


k)
x(t)
y(t)
t
t


l)
x(t)
y(t)
t
t

TEACHING GUIDE

The Doodle Challenge

Overview:

In this Screenshot Presentation assignment, students test their understanding of the Doodle section of the app Action Grapher by attempting to create a doodle that produces the given x vs. arc length and y vs. arc length graphs.

Common Core State Standards:

- F-IF.4

Encyclopedia of Algebraic Thinking:

- Analysis of Change: Interpreting Graphs

Description:

The uniqueness of the Doodle section of the Action Grapher app is that is produces x and y graphs that are a function not of time but of arc length. This will force students to take a conceptual leap into a representation that they likely will not be familiar with. By challenging students to draw two-dimensional doodles that produce the given graphs, they will develop an understanding of the mapping of one representation to the other.

Extensions:

- The Doodle Duel
- Game Two of the Doodle section of Action Grapher

The Doodle Duel

For use with the iPad app *Action Grapher*

Center for Algebraic Thinking

Name: Date: Period:

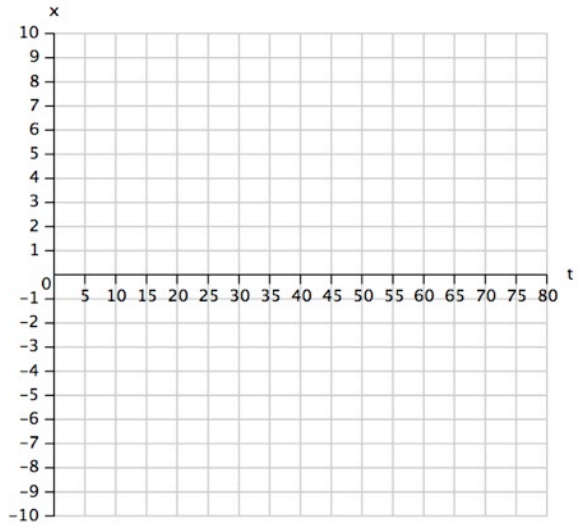
With a partner, try out this two-person challenge:

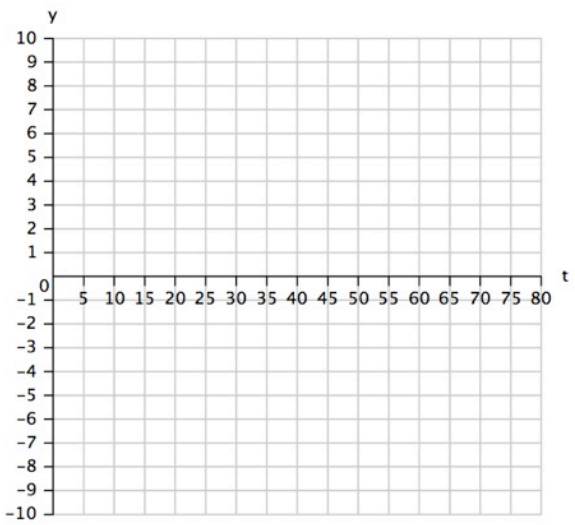
- Step 1: Player one draws a doodle that is no longer than the agreed upon length limit and then sketches the x vs arc length and y vs. arc length in the space provided below.
- Step 2: Player one clears the doodle from the screen, then hands the iPad and the sketched graphs to player two.
- Step 3: Player two attempts to draw a doodle so that the graphs match those produced by player one. If player two succeeds within the agreed upon time limit, then player two gets a point. If not, player one gets a point. You can keep track of points using tallies in the space provided below.
- Step 4: Players switch roles and then repeat Steps 1 through 3

Length Limit: _____ Time Limit: _____

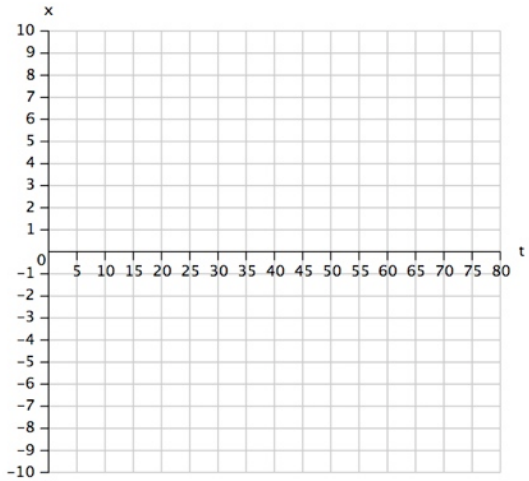
Player One Score: _____ Player Two Score: _____

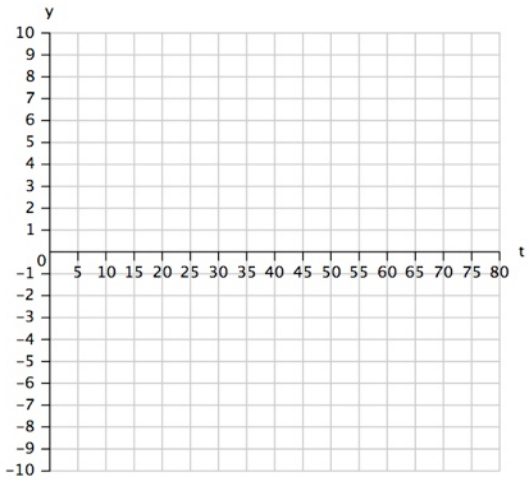
Player One Doodle:



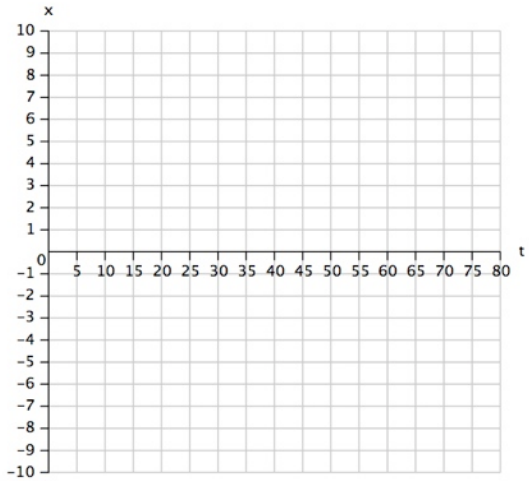


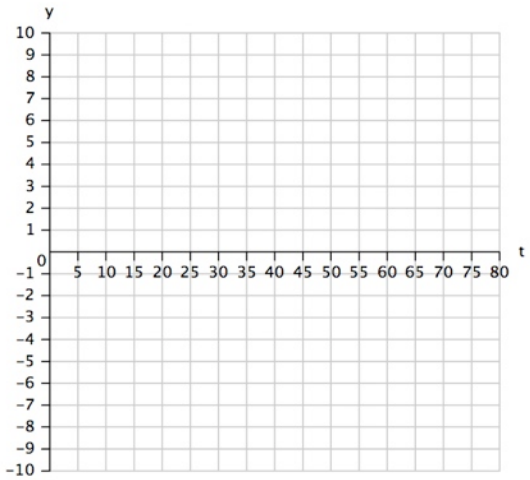
Player Two Doodle:



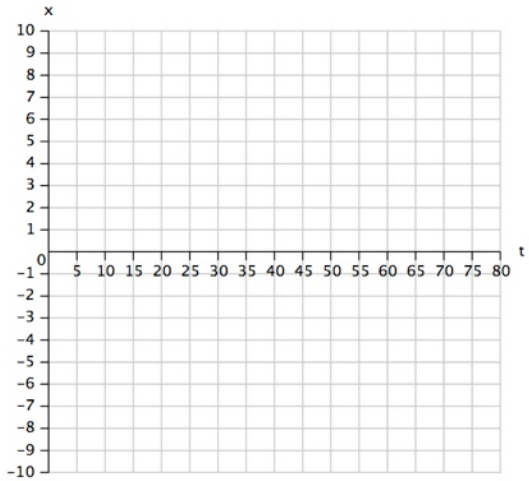


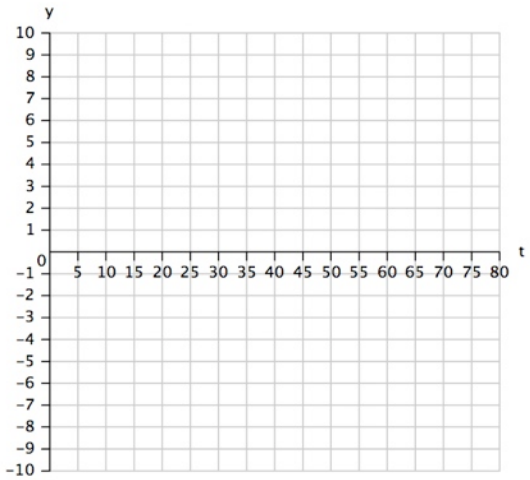
Player One Doodle:





Player Two Doodle:





TEACHING GUIDE

The Doodle Duel

Overview:

Students take turns drawing a doodle, recording the x vs. arc length and y vs. arc length, and then challenging the other student to reproduce their doodle from just the x vs. arc length and y vs. arc length graphs.

Common Core State Standards:

- F-IF.4

Encyclopedia of Algebraic Thinking:

- Analysis of Change: Interpreting Graphs

Description:

In this activity students are given freedom the create their own challenges for each other. Students will need to first familiarize themselves with the Doodle section of the app Action Grapher. The way in which this section of the app takes a two-dimensional doodle and represents that doodle on an x vs. arc length graph and a y vs. arc length graph may be initially confusing to students. For that reason this Doodle Duel activity includes a length limit that the teacher or students must agree upon. You may also want to experiment with setting other limits on the doodles that students draw - such as drawing straight lines only or only drawing certain shapes. By applying the appropriate restrictions on a student-by-student basis, you will be able to provide just the right level of difficulty for each student.

Extensions:

- Doodle Pictionary

Doodle Pictionary

For use with the iPad app *Action Grapher*

Center for Algebraic Thinking

Name: Date: Period:

If this activity, your instructor will present for you an x vs. arc length and y vs. arc length graph. Your task is to determine the object that results from the doodle that produces those two graphs. Use this sheet to record the object for each round:

Round 1: _____ Round 2: _____

Round 3: _____ Round 4: _____

Round 5: _____ Round 6: _____

Round 7: _____ Round 8: _____

Round 9: _____ Round 10: _____

Round 11: _____ Round 12: _____

Round 13: _____ Round 14: _____

TEACHING GUIDE

Doodle Pictionary

Center for Algebraic Thinking

Overview:

Students play a variation of the game pictionary as they practice transitioning from x vs. arc length and y vs. arc length graphs to a complete two dimensional doodle.

Common Core State Standards:

- F-IF.4

Encyclopedia of Algebraic Thinking:

- Analysis of Change: Interpreting Graphs

Description:

This activity adds some playfulness to the process of moving between different graphical representations of the same information. Students should be familiar with the basic process by which the doodle is created from the x vs. t and y vs. t graphs prior to participating in this activity.

Teachers are encouraged to create their own objects, sketch (or screen shot) the x vs. t and y vs. t graphs, and then use those graphs to present to students in this game. To get you started, see the following example graphs that fall within the topic 'shapes':

- Doodle Pictionary: Shapes

Key: 1) Square; 2) Triangle; 3) Rectangle; 4) Circle; 5) Diamond; 6) Ellipse; 7) Parallelogram; 8) Pentagon.

Extensions:

Let students challenge each other by creating their own doodles of objects and then swapping the x vs. t and y vs. t graphs and see if they can identify the original object.

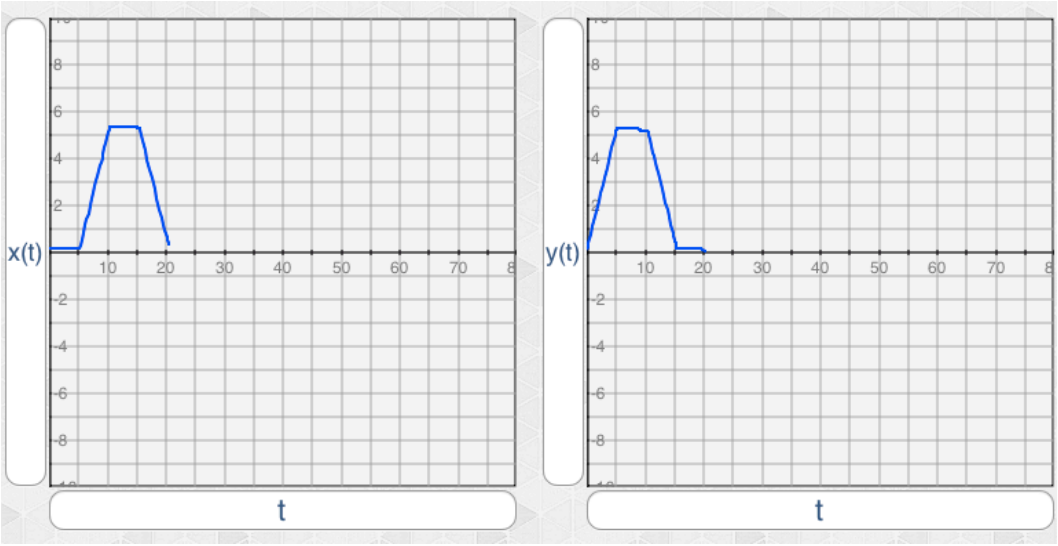
TEACHING RESOURCE

Doodle Pictionary: Shapes

Center for Algebraic Thinking

Present each set of x vs. arc length and y vs. arc length graphs to students and challenge them to identify the shape that results from the doodle that produces those graphs.

1.



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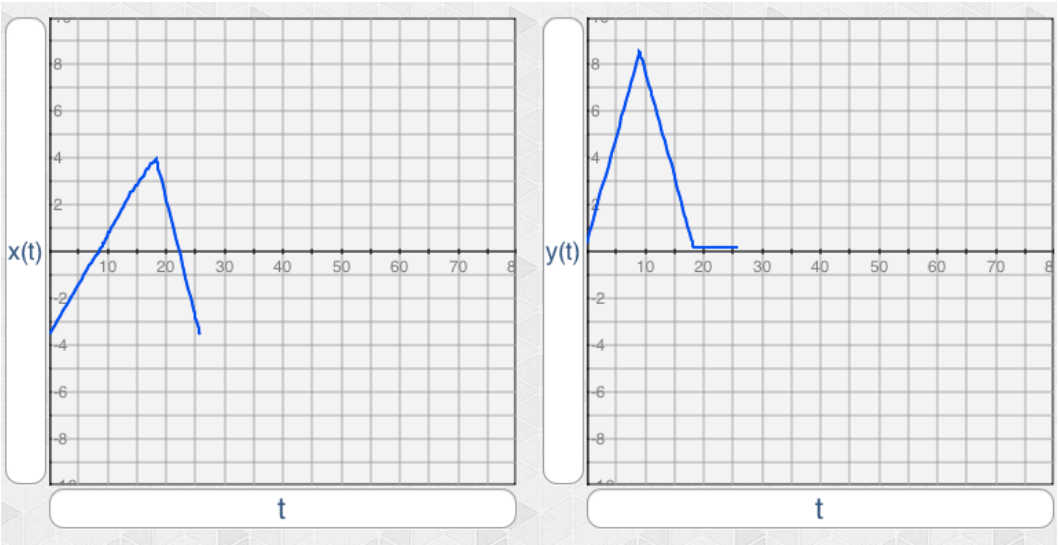

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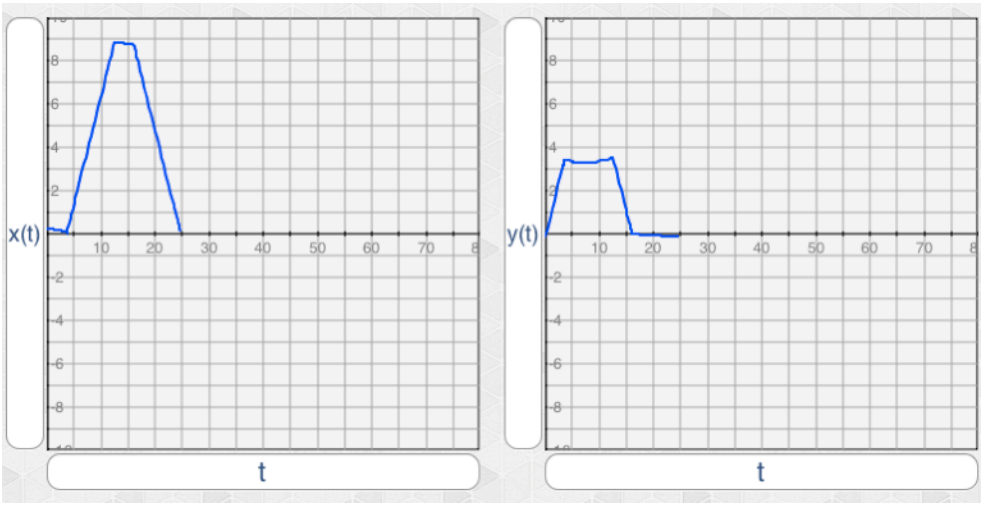

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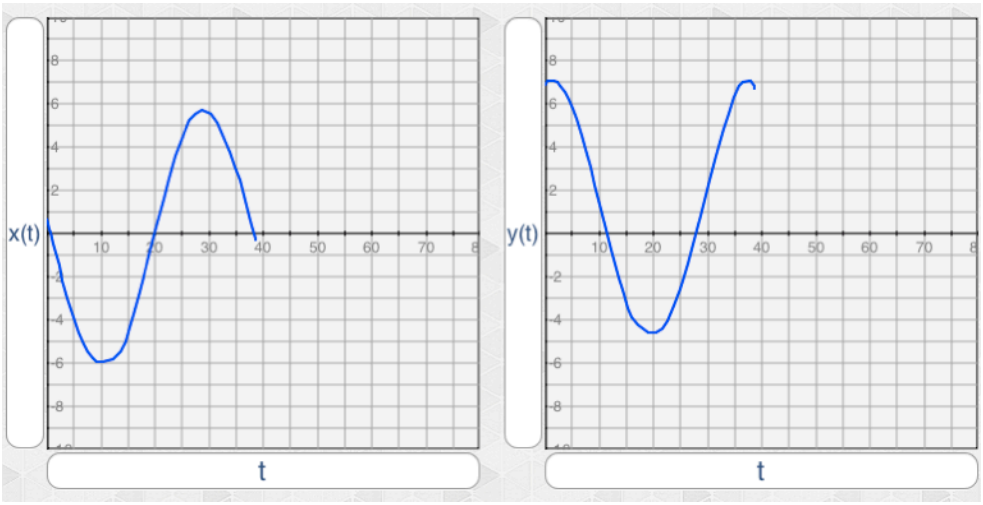
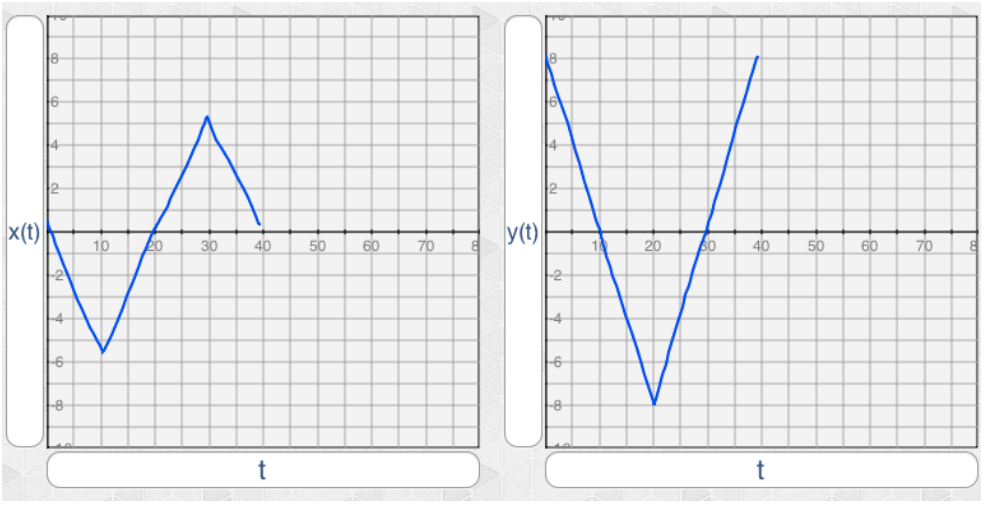


5.



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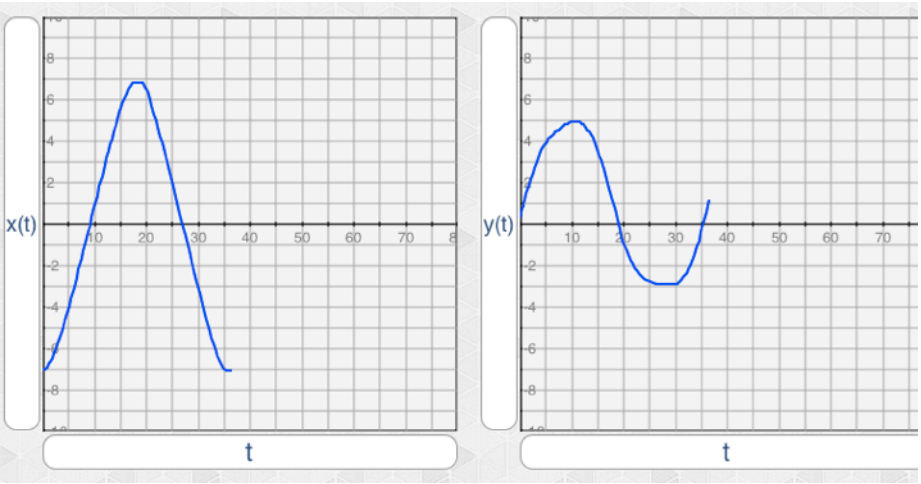
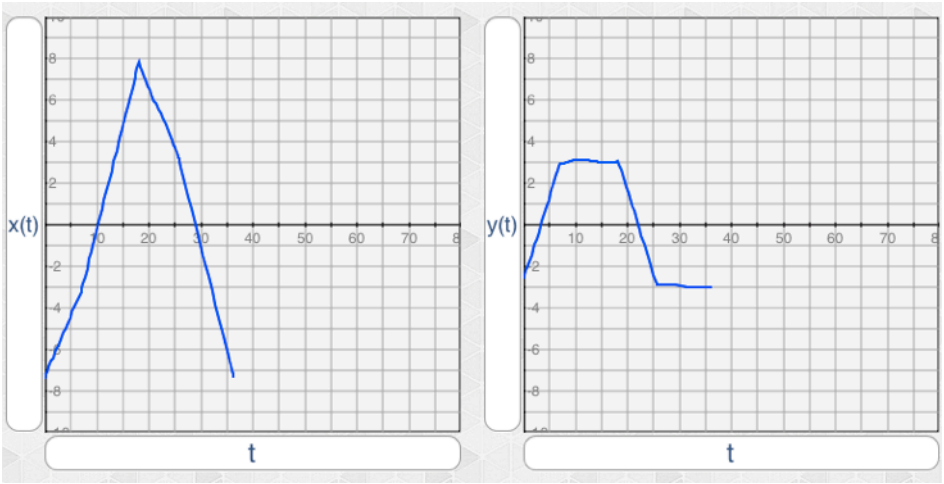
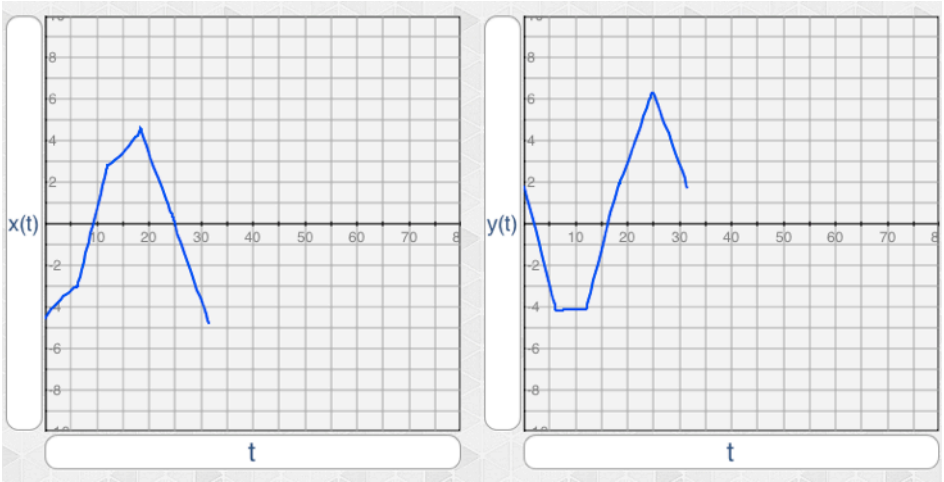


7.



8.



Two-Player Card Clutter

For use with the iPad app *Algebra Card Clutter*

Center for Algebraic Thinking

Names: Date: Period:

In this activity you and a partner will cooperatively play the game Algebra Card Clutter *on the iPad.*

Getting Started

From the home screen of the app, tap the 'Start' button at the top of the menu. Then select the mode you would like to play. If you have not played this game before, tap 'Full Course.' If you have played it before and feel ready for more of a challenge, tap 'Advanced Course.'

Gameplay

Once the game starts, you and your partner will take turns selecting the lowest card on the screen. When it is your turn, your first attempt has to be on your own (without help from your parter). But if you select the wrong card on your first try, then you and your parter can work together to select the correct card. Continue alternating turns throughout the game.

If the time ever runs out on you, then use the space below to record the correct solution for that level, and explain the trouble you had completing the level. Then tap 'Retry' and continue playing. Follow that process until you have completed all 14 levels, and then record you final time at the bottom on the next page.

Level: Solution:

Explanation of the trouble you had with this level:

Level: Solution:

Explanation of the trouble you had with this level:

Level: Solution:

Explanation of the trouble you had with this level:

Level: Solution:

Explanation of the trouble you had with this level:

Continue playing until you finish all 14 levels, and then record your final time below.

Final Time: _____

TEACHING GUIDE

Two-Player Card Cutter

Center for Algebraic Thinking

Overview:

In this activity students cooperatively play the game *Algebra Card Clutter* in which they must select the lowest card on the screen. The numbers on the cards are given in a variety of formats including positive and negative integers, decimals, fractions, absolute values, square roots, and exponentials.

Common Core State Standards:

- 8.NS.A.2

Encyclopedia of Algebraic Thinking:

- Coming soon...

Description:

The purpose of this resources is to turn the engaging and challenging app *Algebra Card Clutter* into a two-player game in order to tap into the social aspect of learning and provide a structured environment for students to help each other when they run into problems that they cannot solve individually.

In this activity students alternate turns, with a key factor being that on each turn the student must make the first attempt on their own. Only if the students gets that first attempt wrong can the second student jump in to help. The idea is that those initial errors will lead to a discussion about the correct answer, and that together (or at times with the instructor's help as well) they will be able to figure out how to move forward and fill in the gaps that made them unable to complete the challenge on their own. Since the students are recording the levels they have trouble with, and explaining the troubles they had, they will be more likely to remember what they learned from each mistake. Teachers can also use the record to get a quick feel for what students are struggling with the most.

Extensions:

- Card Clutter Progress Monitor
- Student-Created Card Clutter

Card Clutter Progress Monitor

For use with the iPad app *Algebra Card Clutter*

Center for Algebraic Thinking

Name: Date: Period:

In this activity you will record your progress as your work through the game Algebra Card Cutter

Getting Started

From the home screen of the app, tap the 'Start' button at the top of the menu. Then select the mode you would like to play. If you have not played this game before, tap 'Full Course.' If you have played it before and feel ready for more of a challenge, tap 'Advanced Course.'

Gameplay

Your task in this game is to tap the cards on the screen from the smallest card value to the largest. When you make an error then some time will be added to your score. If the time ever runs out on you, then use the space below to record the correct solution for that level, and explain the trouble you had completing the level. Then tap 'Retry' and continue playing. Follow that process until you have completed all 14 levels, and then record you final time at the bottom on the next page.

Level: Solution:

Explanation of the trouble you had with this level:

Level: Solution:

Explanation of the trouble you had with this level:

Level: Solution:

Explanation of the trouble you had with this level:

Level: Solution:

Explanation of the trouble you had with this level:

Level: Solution:

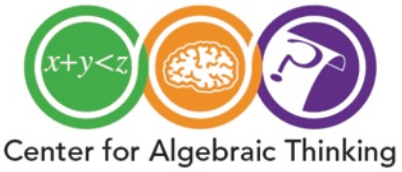
Explanation of the trouble you had with this level:

Continue playing until you finish all 14 levels, and then record your final time below.

Final Time: _____

TEACHING GUIDE

Card Clutter Progress Monitor



Overview:

In this activity students work through the game *Algebra Card Clutter* in which they must tap cards from the smallest value to the largest in progressively harder levels. With this paper resource students record and reflect on any situations in which they are not able to pass a level on their own.

Common Core State Standards:

- 8.NS.A.2

Encyclopedia of Algebraic Thinking:

- Coming soon...

Description:

The strength of this app is that the difficulty builds in a way that makes it accessible to students with varying degrees of prior knowledge. The form of the numbers on the cards progress from integers, to fractions, to decimals, to absolute values, to exponents, and finally to square roots. Once a student runs into a topic that is difficult enough that they cannot pass the level, then this resource provides the space to stop and reflect on that topic, and get additional help if needed. They record the correct order of the cards (which is presented to them within the app) and then they write about the trouble they had with that level. This reflection will both help them to work through confusions on their own as well as solidify the new knowledge.

Extensions:

- Student-Created Card Clutter

Student-Created Card Clutter

For use with the iPad app *Algebra Card Clutter*

Center for Algebraic Thinking

Name: Date: Period:

In the iPad app Algebra Card Clutter *your task is to sort cards from the smallest number to the largest number. In this activity you and your classmates will* ***create your own cards****, pool those cards together, and then play the same game hands-on.*

Step 1: Card Creation

The first step is to create the physical cards that you and your classmates will use to play the game. You need to create a total of 12 cards, two in each of the categories given below. At least five of your cards need to have negative values. Here are the categories:

Integers	Decimals	Exponentials
Fractions	Absolute Values	Square Roots

So, create 12 cards total, two from each category above, and remember that at least five of your cards need to have negative values.

Step 2: Combining Cards

Now you will combine the cards you created with the cards of your classmates. The idea here to to create a large stack of cards that your group will be able to use to play the game. So pool all of your cards together, either all of those within your table group, or all of those within your entire class (depending on the directions from your instructor).

Step 3: Play the Game!

After mixing all of the cards together, and then redistributing them to the different groups, you will starting playing the game. In groups of two, start with your stack of cards face down on the table in front of you. You and your partner will take turns. One player starts by selecting the first five cards from the deck and placing them face up on the desk. That player then has a maximum time of **one minute** to order those cards from the smallest value to the largest value. Once that player is satisfied with the order (or after a minute has passed) you will record *the decimal values* of each of those cards on the following page in order to see if that player ordered the cards correctly. You can use a calculator for this step, but you cannot use a calculator when doing the ordering. After recording the decimal values, check the appropriate blank to show whether or not the order was correct. The second player then follows the same process. Continuing going back and forth and recording the results. If you run out of cards, you can reshuffle your deck and continue playing.

Round One

Player 1: _____ _____ _____ _____ _____ Correct Order? Y ___ N ___

Player 2: _____ _____ _____ _____ _____ Correct Order? Y ___ N ___

Round Two

Player 1: _____ _____ _____ _____ _____ Correct Order? Y ___ N ___

Player 2: _____ _____ _____ _____ _____ Correct Order? Y ___ N ___

Round Three

Player 1: _____ _____ _____ _____ _____ Correct Order? Y ___ N ___

Player 2: _____ _____ _____ _____ _____ Correct Order? Y ___ N ___

Round Four

Player 1: _____ _____ _____ _____ _____ Correct Order? Y ___ N ___

Player 2: _____ _____ _____ _____ _____ Correct Order? Y ___ N ___

Round Five

Player 1: _____ _____ _____ _____ _____ Correct Order? Y ___ N ___

Player 2: _____ _____ _____ _____ _____ Correct Order? Y ___ N ___

Round Six

Player 1: _____ _____ _____ _____ _____ Correct Order? Y ___ N ___

Player 2: _____ _____ _____ _____ _____ Correct Order? Y ___ N ___

Round Seven

Player 1: _____ _____ _____ _____ _____ Correct Order? Y ___ N ___

Player 2: _____ _____ _____ _____ _____ Correct Order? Y ___ N ___

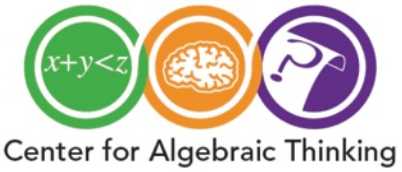
Round Eight

Player 1: _____ _____ _____ _____ _____ Correct Order? Y ___ N ___

Player 2: _____ _____ _____ _____ _____ Correct Order? Y ___ N ___

TEACHING GUIDE

Student-Created Card Clutter



Overview:

In this activity students bring the game Algebra Card Clutter into the real world by making their own physical cards and then playing the game hands-on with classmates.

Common Core State Standards:

- 8.NS.A.2

Encyclopedia of Algebraic Thinking:

- Coming soon...

Description:

After bring introduced to the idea of this game via the iPad app *Algebra Card Clutter*, this activity will bring it into the real world and give students ownership of it.

You begin by having each student create their own cards. Each card has just one number on it, and students must create a total of 12, with two numbers in each of the following categories: integers, fractions, decimals, absolute values, exponentials, and square roots. By requiring that students create two cards for each category, it forces students to write numbers in forms they might not normally use. Depending on students prior knowledge, teachers can adjust this step by including only those categories with which students are comfortable. To ensure that students explore the entire number line, a second requirement is that at least five of the cards contain negative values.

You then pool all of the cards together, mix them, and then redistribute them to groups of two. This mixing and redistributing means that students will get many cards that they did not create, but were instead created by classmates, which adds some variability into the values that students will be playing with.

In groups of two, students then use their stack of cards to play the game, in which players alternate selecting five cards from the deck and then ordering those cards from the smallest value to the largest. To check their ordering they convert each value to a decimal and record those decimals on a separate piece of paper. This record provides a means for accountability - the teacher can collect this at the end of the activity.

Extensions:

You can extend this activity by varying the categories used (leaning toward either the easier categories or more difficult ones). You can also vary the number of cards created by each student, and the number of cards selected from the deck for each round of play.

Sticky Note Cover Up Math

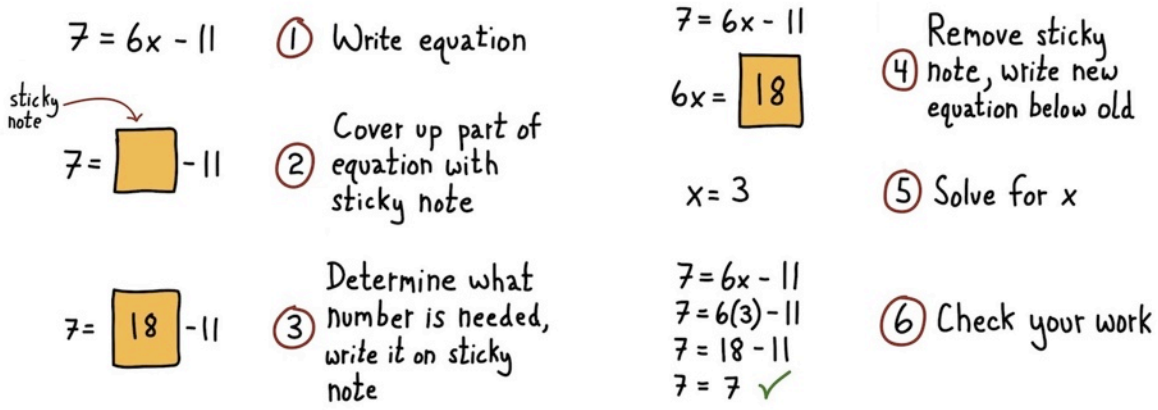
For use with the iPad app *Cover Up*

Center for Algebraic Thinking

Name: Date: Period:

In this activity you will learn how to use sticky notes to cover up certain parts of an equation so that you can focus in on the solution one step at a time. After learning the process you will get to practice your new skills with the iPad app *Cover Up*.

Here's how the process works. Let's say you are given the equation **7 = 6x - 11** and your task is to solve for x. You can use sticky notes to help you in the following way:



Using the cover up method allows you to get started on a problem by ignoring the complicated part of an equation - the part with the variable. If you look at the equation as it is written in Step 1, you might not know where to start. But once you cover up the **6x** with a sticky note, it looks a whole lot more approachable. Now you just have to figure out what number minus 11 equals 7. A little mental math tells you that number is 18. So now you know that whatever is behind the sticky note has to be equal to 18. The last step is to figure out what number times 6 equals 18. A bit more mental math will tell you that the answer is 3. A final check of your work confirms that your answer is correct.

Use the space below to give this method a try with the following equation: **15 = 4x + 3**

TEACHING GUIDE

Sticky Note Cover Up Math

Center for Algebraic Thinking

Overview:

In this activity the student is introduced to the Cover Up Math Method with a detailed example and full explanation, and then given a chance to practice the method on a new problem.

Common Core State Standards:

- A-SSE.1
- A-REI.1

Encyclopedia of Algebraic Thinking:

- Algebraic Relations: Equations Involving Negatives
- Algebraic Relations: Flexible Use of Solution Strategies
- Algebraic Relations: Student Intuition and Informal Procedures
- Variables and Expressions: Letter Used as a Specific Unknown

Description:

This activity is designed as an introduction to the Cover Up Math Method. The purpose is to explain the process of the method prior to letting students practice the method with the iPad app *Cover Up*. If students jump right into using the app without an explanation of the method, they will be less likely to transfer the skills when working on problems on their own. One approach to this activity is to have students work through it individually. Another option is to hand out the activity and then work through the example as a class, using an actual sticky note on the board or projected from a doc cam. By introducing this method with pen and paper first, and then following up with digital practice (see the extension below), students will be better prepared to use it when then return to pen and paper work.

Extensions:

- Cover Up Math App Progress Monitor

Cover Up Math App
Progress Monitor

For use with the iPad app *Cover Up*

Center for Algebraic Thinking

Name: Date: Period:

Use this sheet to record the results obtained using the Cover Up *app and to keep track of the levels and problems that you have completed.*

Level	Equation	Solution

TEACHING GUIDE

Cover Up Math App Progress Monitor

Overview:

This resource allows students to keep track of their progress as they work through the various levels of the app *Cover Up*.

Common Core State Standards:

- A-SSE.1
- A-REI.1

Encyclopedia of Algebraic Thinking:

- Algebraic Relations: Equations Involving Negatives
- Algebraic Relations: Flexible Use of Solution Strategies
- Algebraic Relations: Student Intuition and Informal Procedures
- Variables and Expressions: Letter Used as a Specific Unknown

Description:

This resource serves two purposes. First, it provides students the space to monitor their own progress through the app while simultaneously bridging the gap between digital tools and pencil and paper work. Second, it provides teachers with a record of the work that students have completed related to the app. By including sections for the student to record each problem as well as the solution, it helps to ensure that each student is actually completing the problems put forward by the app.

Extensions:

- Sticky Note Cover Up Challenge

Sticky Note Cover Up Challenge

For use with the iPad app *Cover Up*

Center for Algebraic Thinking

On a separate piece of paper, use the cover up math method to solve the equations below. If you need a refresher, check out the app Cover Up*:*

Level 1:

1) 5x + 7 = 32
2) 12 - 3x = 6
3) 9 = 25 - 4x
4) 23 + 7x = 9
5) 8x - 13 = 35
6) 19 + 3x = 4
7) -1 + 7x = -22
8) 5 - 2x = -21
9) -17 = 5 - 11x
10) -8 + 9x = 46

Level 2:

1) 3(x + 5) = 24
2) 5(x - 2) = -25
3) 13 = 2(x + 3)
4) 7 + x/3 = 11
5) 9 = x/5 - 4
6) -4(x - 7) = 16
7) 12(x - 7) = 108
8) -8 = 2(x - 5)
9) 11 - x/3 = 4
10) -15 = x/9 - 13

Level 3:

1) 3 = (18 - x)/4
2) 6 = 42/(x - 5)
3) 24/(2x) = 4
4) 8 = (1/2)(x + 3)
5) (5x)/4 - 3 = 7
6) -11 = (9 + x)/2
7) -3 = 18/(x - 2)
8) 49/(4x) = -7
9) -9 = (1/3)(x - 15)
10) (-8x)/2 + 5 = -7

Level 4:

1) $(3x + 4)/2 = 11$
2) $5(x + 2) + 7 = 37$
3) $(8x + 3)/3 - 5 = 4$
4) $9 = (7 - 3x)/4 + 6$
5) $11 = 55/(7x - 16)$
6) $(4x - 6)/3 = -10$
7) $-6(x + 5) - 3 = -57$
8) $(11x - 10)/ 7 + 23 = 31$
9) $-1 = (-3 + 6x)/3 - 12$
10) $7 = 98/(-4x + 2)$

Level 5:

1) $2(x^2 + 3) - 7 = 49$
2) $13 - 24/(x + 3) = 9$
3) $5(4x - 13)^3 = 135$
4) $81/(x-5)^2 + 17 = 26$
5) $7 + (x + 2)^2 = 71$
6) $6(x^2 - 27) + 15 = -51$
7) $73 - 42/(x - 12) = 67$
8) $13(9x - 16)^3 = 104$
9) $112/(x-6)^2 - 5 = 23$
10) $-15 + (x - 9)^2 = 34$

TEACHING GUIDE

Sticky Note Cover Up Challenge

Center for Algebraic Thinking

Overview:

Students practice the Cover Up Math Method with pencil and paper problems that mirror the problems encountered in the iPad app *Cover Up*.

Common Core State Standards:

- A-SSE.1
- A-REI.1

Encyclopedia of Algebraic Thinking:

- Algebraic Relations: Equations Involving Negatives
- Algebraic Relations: Flexible Use of Solution Strategies
- Algebraic Relations: Student Intuition and Informal Procedures
- Variables and Expressions: Letter Used as a Specific Unknown

Description:

This activity is designed as a follow up to the app *Cover Up*. The purpose is to help students transfer the skills they learned while using the app to a pencil and paper activity so that they will then be more likely to use the method effectively when working on their own problems.

Key:

Level 1 - **1**: 5; **2**: 2; **3**: 4; **4**: -2; **5**: 6; **6**: -5; **7**: -3; **8**: 13; **9**: 2; **10**: 6
Level 2 - **1**: 3; **2**: -3; **3**: 3.5; **4**: 12; **5**: 65; **6**: 3; **7**: 16; **8**: 1; **9**: 21; **10**: -18
Level 3 - **1**: 6; **2**: 12; **3**: 3; **4**: 13; **5**: 8; **6**: -31; **7**: -4; **8**: -7/4; **9**: -12; **10**: 3
Level 4 - **1**: 6; **2**: 4; **3**: 3; **4**: -5/3; **5**: 3; **6**: -6; **7**: 4; **8**: 6; **9**: 6; **10**: -3
Level 5 - **1**: 5; **2**: 3; **3**: 4; **4**: 8; **5**: 6; **6**: 4; **7**: 19; **8**: 2; **9**: 8; **10**: 16

Extensions:

- Comparison of Methods: Cover Up VS Algorithmic
- Mental Math Cover Up Challenge

Comparison of Methods:
Cover Up VS Algorithmic

For use with the iPad app *Cover Up*

Center for Algebraic Thinking

Name: Date: Period:

In this assignment you will compare two methods of solving equations: the cover up method and the algorithmic method.

Using the Two Methods

Solve the following problems using the **algorithmic** method:

1) $5 - 2x = -21$

2) $-11 = (9 + x)/2$

Now solve these problems using the **cover up** method:

3) $19 + 3x = 4$

4) $-3 = 18/(x + 2)$

Reflecting on the Two Methods

What is different about these two methods?

Which of the two methods do you prefer using? Explain your choice.

TEACHING GUIDE

Comparison of Methods: Cover Up VS Algorithmic

Overview:

In this activity students solve a few equations using the cover up method and a few using the algorithmic method, and then reflect on the differences between the two methods.

Common Core State Standards:

- A-SSE.1
- A-REI.1

Encyclopedia of Algebraic Thinking:

- Algebraic Relations: Equations Involving Negatives
- Algebraic Relations: Flexible Use of Solution Strategies
- Algebraic Relations: Student Intuition and Informal Procedures
- Variables and Expressions: Letter Used as a Specific Unknown

Description:

The purpose of this activity is to give students the opportunity to reflect on two different methods for solving equations. When we refer to the "algorithmic method" we are referring to the step by step process of solving an equation by executing identical operations on both sides of the equal sign: (from problem 1 in this activity) subtract 5 from both sides to isolate the term with x in it, then divide both sides by -2 to get x = 13. When we refer to the "cover up method," we are referring to the method introduced in the activity Sticky Note Cover Up Math that goes like this: (from problem 3 in this activity) in order to get 4 from 19 I have to subtract 15, then in order to get 15 from 3 I have to multiply by 5. Both methods have their merits, and the purpose of this activity is to point out those merits and give students multiple approaches to solving equations.

Extensions:

- The Mental Math Cover Up Challenge

The Mental Math Cover Up Challenge

For use with the iPad app *Cover Up*

Center for Algebraic Thinking

Name: Date: Period:

In this activity you will test you mental math skills by seeing how quickly you can use the cover up math method to solve equations without writing anything down!

Use the cover up math method **in your head** to solve the following problems:

Level 1:

1) 5x + 7 = 32
2) 12 - 3x = 6
3) 9 = 25 - 4x
4) 23 + 7x = 9
5) 8x - 13 = 35
6) 19 + 3x = 4
7) -1 + 7x = -22
8) 5 - 2x = -21
9) -17 = 5 - 11x
10) -8 + 9x = 46

Level 2:

1) 3(x + 5) = 24
2) 5(x - 2) = -25
3) 13 = 2(x + 3)
4) 7 + x/3 = 11
5) 9 = x/5 - 4
6) -4(x - 7) = 16
7) 12(x - 7) = 108
8) -8 = 2(x - 5)
9) 11 - x/3 = 4
10) -15 = x/9 - 13

Level 3:

1) $3 = (18 - x)/4$
2) $6 = 42/(x - 5)$
3) $24/(2x) = 4$
4) $8 = (1/2)(x + 3)$
5) $(5x)/4 - 3 = 7$
6) $-11 = (9 + x)/2$
7) $-3 = 18/(x - 2)$
8) $49/(4x) = -7$
9) $-9 = (1/3)(x - 15)$
10) $(-8x)/2 + 5 = -7$

Level 4:

1) $(3x + 4)/2 = 11$
2) $5(x + 2) + 7 = 37$
3) $(8x + 3)/3 - 5 = 4$
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10) $7 = 98/(-4x + 2)$

Level 5:

1) $2(x^2 + 3) - 7 = 49$
2) $13 - 24/(x + 3) = 9$
3) $5(4x - 13)^3 = 135$
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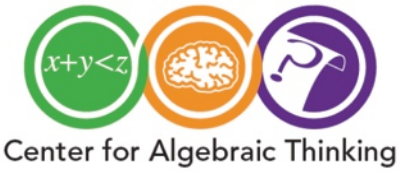
6) $6(x^2 - 27) + 15 = -51$
7) $73 - 42/(x - 12) = 67$
8) $13(9x - 16)^3 = 104$
9) $112/(x-6)^2 - 5 = 23$
10) $-15 + (x - 9)^2 = 34$

TEACHING GUIDE

The Mental Math Cover Up Challenge



Overview:

In this activity students are challenged to see if they can use the cover up math method to solve equations in their head.

Common Core State Standards:

- A-SSE.1
- A-REI.1

Encyclopedia of Algebraic Thinking:

- Algebraic Relations: Equations Involving Negatives
- Algebraic Relations: Flexible Use of Solution Strategies
- Algebraic Relations: Student Intuition and Informal Procedures
- Variables and Expressions: Letter Used as a Specific Unknown

Description:

Once students have practiced using the cover up math method with pencil, paper, and sticky notes, then they might be up for the challenge of solving equations in their head using the same method. This activity gives them the opportunity to take on that challenge. Note that the equations in this activity are the same as those in the activity Sticky Note Cover Up Challenge.

Extensions:

If students are enjoying the challenge of using the cover up math method to solve equations in their head, then you could add some friendly competition into the mix by holding a class tournament to see who is the fastest at solving equations using mental math. While two students at a time go head-to-head, the rest of the class could also participate by recording their answers on a sheet of paper (whereas the competitors would share their answers vocally - maybe by buzzing in first).

Deriving the Diamond Method:
Part 1

For use with the iPad app *Diamond Factor*

Center for Algebraic Thinking

Name: Date: Period:

In this assignment you will learn about the Diamond Method of factoring quadratic equations. After you learn this method you will get to practice it using the iPad app Diamond Factor*.*

1. Use the Distributive Property or the FOIL method to show that $(x + 3)(x + 2) = x^2 + 5x + 6$:

   Note: You may not need all of the lines provided

   $(x + 3)(x + 2)$ = _____

   = _____

   = _____

   = $x^2 + 5x + 6$

2. The goal of **factoring** is to take an expression such as $x^2 + 5x + 6$ and "reverse distribute" it to get to $(x + 3)(x + 2)$. Your next task is to develop a method that will allow you to factor expressions in that way. The key is finding the connection between the bold terms on the left side of the equation and the bold terms on the right side of the equation:

   $$(x + \mathbf{3})(x + \mathbf{2}) = x^2 + \mathbf{5}x + \mathbf{6}$$

   What connection do you see between the two bold numbers on the left and the two bold numbers on the right of the equation above?

3. Now it's time to test the connection that you developed. See if it holds true for the following equations. If it doesn't, keep modifying the connection until it works for ALL of the equations.

   a. $(x + \mathbf{4})(x + \mathbf{1}) = x^2 + \mathbf{5}x + \mathbf{4}$

   b. $(x + \mathbf{2})(x + \mathbf{-3}) = x^2 + \mathbf{-1}x + \mathbf{-6}$

   c. $(x - \mathbf{6})(x + \mathbf{4}) = x^2 - \mathbf{2}x - \mathbf{24}$

   d. $(x - \mathbf{7})(x - \mathbf{2}) = x^2 - \mathbf{9} + \mathbf{14}$

TEACHING GUIDE

Deriving the Diamond Method: Part 1

Overview:

In this activity students set the stage for learning the Diamond Method as they discover for themselves the connection between the factored form and foiled form of a quadratic equation.

Common Core State Standards:

- A-SSE.3
- F-IF.8
- Standard for Mathematical Practice #7

Encyclopedia of Algebraic Thinking:

- Patterns and Functions

Description:

The purpose of this assignment is both to remind students about FOILing (i.e. distributing) a factored equation and to give them the chance to discover for themselves why the diamond method works before they learn the mechanics of that method. This approach is potentially more worthwhile than simply *telling* them what the diamond method is because if they know where the method comes from then they will see that tools like these have a logical origin that they themselves can often discover. Students will also be more likely to remember the method after having seen where it comes from.

Extensions:

- Discovering the Diamond Method: Part 2

Deriving the Diamond Method:
Part 2

For use with the iPad app *Diamond Factor*

Center for Algebraic Thinking

Name: Date: Period:

In Part 2 of this assignment you will take the connection that you developed in Part 1 and solidify that connection using the Diamond Method.

In Part 1 you should have found that to factor an equation such as $x^2 + 5x + 6$, you need to find two numbers that add to 5 and multiply to 6. The two numbers that satisfy those conditions are 2 and 3, so: $x^2 + 5x + 6 = (x + 2)(x + 3)$.

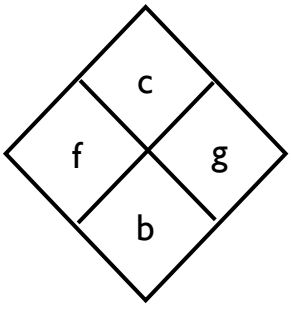
We can *generalize* that relationship in the following way: If we have an equation of the form

$$x^2 + bx + c$$

Then in order to factor that expression we need to find two numbers that add together to get b and multiply together to get c. Let's call the two numbers that meet those conditions f and g. What you found out in Part 1 of this assignment is that

IF
$f + g = b$ and $f*g = c$
THEN
$x^2 + bx + c = (x + f)(x + g)$

To help us remember that relationship and use it to solve problems, we have the Diamond Method, which works like this:

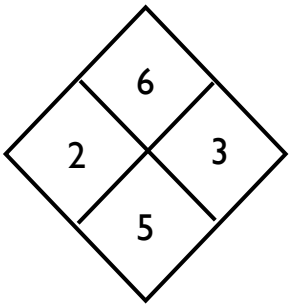


The two numbers in the left and right sections of the diamond have to add together to get the number in the bottom sections of the diamond and multiply together to get the number in the top section of the diamond. Check out the next page for a few examples.

EXAMPLES:

Ex 1.

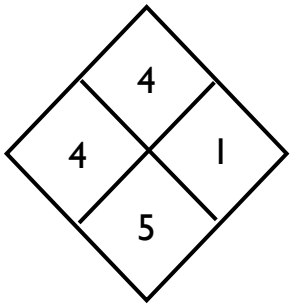
$x^2 + 5x + 6$



$(x + 2)(x + 3)$

Ex 2.

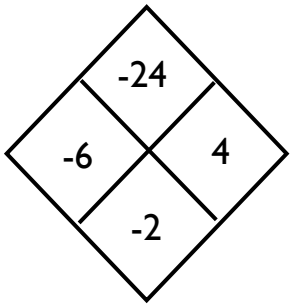
$x^2 + 5x + 4$



$(x + 4)(x + 1)$

Ex 3.

$x^2 - 2x - 24$

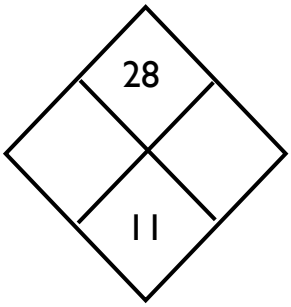


$(x - 6)(x + 4)$

Now it's time for you to get some practice using this method. See if you can fill in the missing sections of these diamonds, and then write in the factors below the diamond:

1.

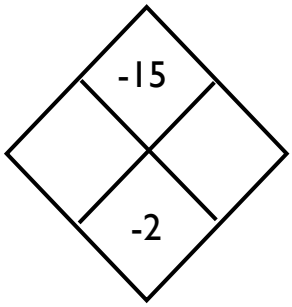
$x^2 + 11x + 28$



(x)(x)

2.

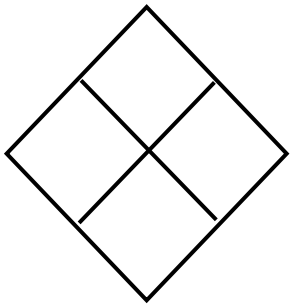
$x^2 - 2x - 15$



(x)(x)

3.

$x^2 + 3x - 54$



(x)(x)

Now you get to practice this new skill using the app *Diamond Factor*. Your goal is to get all the way to Level 5. Open it up and see how you do!

TEACHING GUIDE

Deriving the Diamond Method: Part 2

Overview:

In this activity students build on the foundation set in Part 1 as they learn the mechanics of the Diamond Method and then practice using that method.

Common Core State Standards:

- A-SSE.3
- F-IF.8
- Standard for Mathematical Practice #7

Encyclopedia of Algebraic Thinking:

- Patterns and Functions

Description:

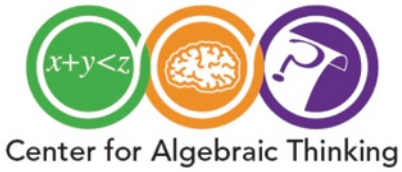
In this follow-up to Part 1, students are walked through the process of *generalizing* the connection that they (hopefully) made between the factored form and distributed form of a quadratic equation. The mechanics of Diamond Method are then introduced, and students get a chance to practice using that method with pencil and paper before taking on the challenges in the app *Diamond Factor*.

Extensions:

After learning this method students can work through the "Play" section of the app *Diamond Factor* and see how many levels they can pass through in a given amount of time.

Discovering the Mystery Function

For use with the iPad app *Function Mystery Machine*



For this activity you will use the iPad app *Function Mystery Machine* to test your knowledge of linear funcitons and see if you can outsmart your classmates in the two-player challenge. Complete Phase 1 below before moving to Phase 2.

Phase 1: One-Player Practice

Steps:

1. Work your way up through Level 4, to the start of Level 5.
2. Show the iPad to your instructor to prove that you are ready to move on to Phase 2.

Phase 2: Two-Player Challenge

Steps:

1. From the Menu, Player 1 writes in a linear equation and taps 'Start Two-Player'.
2. Player 2 has 5 attempts to write in the correct equation. Player 2 gains a point from writing in the correct equation, but loses a point if unable to write the correct equation within 5 attempts.
3. Player 2 writes in a linear equation.
4. Player 1 has 5 attempts to write in the correct equation. Player 1 gains a point from writing in the correct equation, but loses a point if unable to write the correct equation within 5 attempts.
5. Continue alternating roles until a player reaches 5 points - this player is the winner!
6. Once a player reaches 5 points, show the iPad to the instructor as a check for participation.

TEACHING GUIDE

Discovering the Mystery Function

Overview:

In this activity students use the iPad app *Function Mystery Machine* to first practice individually the process of identifying a linear equation based on a table of points, and then use the two-player mode to challenge a classmate.

Common Core State Standards:

- F-LE.A.2
- F-BF.A.1b

Encyclopedia of Algebraic Thinking:

- Analysis of Change: Understanding Slope
- Patterns and Functions: Function Machine
- Patterns and Functions: Linear Function

Description:

In this two-phase activity, students first show that they have mastered individually the skill of defining a linear function based on a table of points, then move on to a two-player challenge in which students write equations for each other discover. The strength of this app is that the two-player mode is built into the application, so students need only to follow the prompts provided by the app. The steps listed in this activity are simply an additional reference.

Extensions:

If both students participating in the two-player challenge have completely mastered linear equations, then they can start incorporating quadratic and other complex functions. One-Player Levels 5 and above incorporate more complex functions if students need individual practice first.

Drag and Drop Linear Equations

For use with the iPad app *Linear Model*

Center for Algebraic Thinking

Name: Date: Period:

1. Use the central blue dot on the graph to drag the line around. What happens to the equation as you drag that line around?

2. What is the name of the thing that you are adjusting when you drag that central blue dot?

3. Use one of the outer blue dots to rotate the line. What happens to the equation as you rotate the line?

4. What is the name of the thing that you are adjusting when you rotate the line?

5. Use the drag and drop features of this app to produce lines with each of the following equations:

   a) $y = 2x + 4$ b) $y = -0.5x + 3$ c) $y = 3.25x - 6$

   d) $y = x - 7.33$ e) $y = 0.33x + 5.25$ f) $y = -4x - 2.75$

TEACHING GUIDE

Drag and Drop Linear Equations

Overview:

In this activity students use a tap-and-drag approach to linear equations as they manipulate a line and watch how that changes the equation of that line.

Common Core State Standards:

- F-LE.A.2

Encyclopedia of Algebraic Thinking:

- Analysis of Change: Understanding Slope
- Analysis of Change: Understanding the “b” in “y = mx + b”
- Patterns and Functions: Linear Function

Description:

The value of the app *Linear Model* lies in the ease with which students can manipulate a line simply by tapping and dragging blue dots that either translate or rotate the line. Because they can see corresponding changes in the equation for that line, as well as changes in a table of values, this application makes for a useful exploration tool. This activity allows students to become familiar with the ways in which they can manipulate a linear equation, and gives students a chance to connect those variations to specific vocabulary (y-intercept and slope).

Extensions:

- Multiple Representations of Lines

Multiple Representations of Lines

For use with the iPad app *Linear Model*

Center for Algebraic Thinking

Name: Date: Period:

In the "Function:" text input section, type in **y = 2x - 3**.

1. How many complete representations of this linear equation are there on the screen?

2. Describe each representation:

3. How can you use the table of x and y coordinate points to determine the slope of the line? Give an example using the equation above.

4. How can you use the equation to determine the slope and y-intercept of the line?

TEACHING GUIDE

Multiple Representations of Lines

Center for Algebraic Thinking

Overview:

In this activity students use the iPad app *Linear Model* to explore three different ways of representing a line: as an equation, as a table of values, and as a graph.

Common Core State Standards:

- F-LE.A.2
- F-IF.C.7

Encyclopedia of Algebraic Thinking:

- Analysis of Change: Understanding Slope
- Analysis of Change: Understanding the "b" in "y = mx + b"
- Patterns and Functions: Linear Function

Description:

By exploring linear equations in a variety of ways, students will be better able to jump from one representation to another and gain useful knowledge from each. The ideal situation is for students to be able to start with one representation (either an equation, a table of values, or a graph) and produce the other two on their own. This activity gives students the opportunity to both focus on each representation individually and then make connections between the different representations. By making connections between the three representations, students will develop a fuller and more comprehensive understanding of linear equations.

Extensions:

Since this activity focuses on a single linear equation, teachers can extend the investigation by having students explore other linear equations that the teacher provides or that students themselves come up with.

Line-Building Challenge

For use with the iPad app *Lion Grapher*

Center for Algebraic Thinking

Name: Date: Period:

*This is a **Screenshot Presentation** assignment. Here are your objectives:*

Objective 1

Using the Intercept Mode, create two of the following equations:

y = x	y = 2x + 3	y = -0.5x + 5
y = 1.3x - 7	y = -3x + 1.5	y = -2.25x + 6.5

Objective 2

Using the Point Mode, create two of the following equations:

y = x - 3	y = 7x - 7	y = 0.3x + 2
y = 5.75x - 9	y = -x + 1.5	y = 3x

Objective 3

Using the Line Mode, create two of the following equations:

y = 1.5x	y = 2.25x + 3	y = -0.5x - 8
y = 1.75x - 7	y = -3x + 9	y = 9x + 7

TEACHING GUIDE

Line Building Challenge

Center for Algebraic Thinking

Overview:

In this activity, students use the *Lion Grapher* app to construct lines in three different ways: by entering the y-intercept and a point on the line; by entering two points on the line; and by physically translating and adjusting the slope by touching and dragging.

Common Core State Standards:

- F-LE.A.2
- F-IF.C.7

Encyclopedia of Algebraic Thinking:

- Analysis of Change: Understanding Slope
- Analysis of Change: Understanding the "b" in "y = mx + b"
- Patterns and Functions: Linear Function

Description:

By constructing lines in the three different ways described in the overview above, students will gain a deeper understanding of linear equations. In each mode, students are given six linear equations and must choose two of those to construct. Each mode allows students to focus on a different aspect of a line and its corresponding equation. By providing a variety of linear equations with differing levels of complexity, students with varying degrees of background knowledge will all be able to gain new understanding.

Extensions:

Since the activity asks students to construct two of the given six lines for each of the three objectives, teachers can extend this activity by having student construct all six of the lines for each objective. Students could also move on to the Lion Mode, in which students practice creating linear equations in a game context - they have to construct a line that passes through the head of lion that is placed somewhere on the coordinate system. Students have 200 seconds to complete this task as many times as possible.

Moving from Points to Lines

For use with the iPad app *Point Plotter*

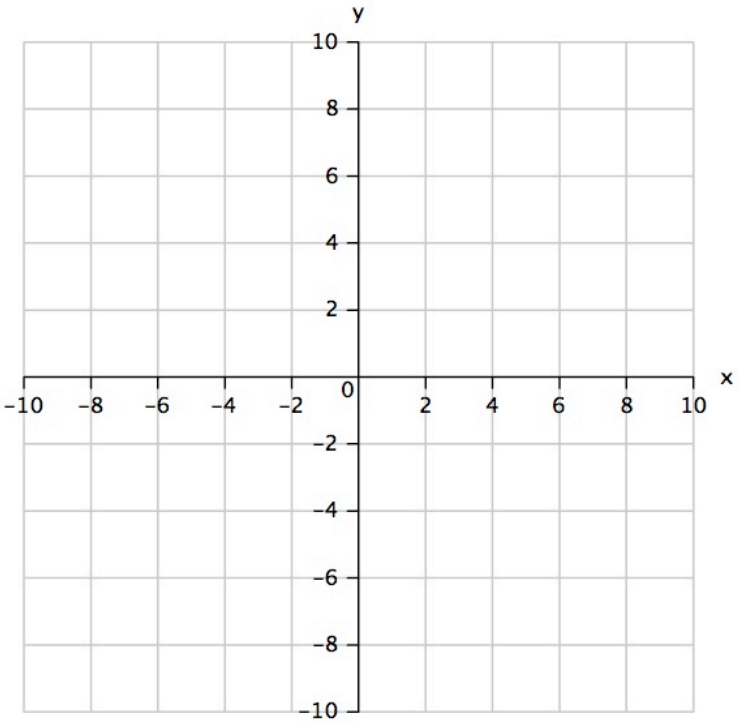
Center for Algebraic Thinking

Name: Date: Period:

The challenge of the iPad app *Point Plotter* is to identify as many points as possible that fall within a particular line segment. In this assignment you will try to figure out how all of those points are related. Once you are skilled enough at *Point Plotter* to fill the "Correct Points" table with at least six points (at any difficulty level), start the investigation below.

Part 1: Graph and Table

1. After completing a *Point Plotter* game during which you identified at least six "Correct Points", graph the line from that game and record the correct points in the table below.



Correct Points (x , y)

Part 2: Searching for a Connection

1. Look at the table of correct points. Search for a connection between those points. Look for consistent changes as you move from one point to the next. In words, describe the connection that you see.

2. In this step you will describe that connection mathematically by trying to come up with an equation that will allow you to determine the y value of a point on the line if you are given the x value of that point. For each attempt below, write an equation for y, then test that equation on three correct points from the table on the previous page. To test your equation, start by copying the x value of a correct point into the x column of the table below, then calculate the y value using the equation you came up with, then add a check mark if the x and y values match the correct point from the previous page. Once you find an equation that works for all of the correct points, move on to the next problem.

Attempt 1: y = _____

x	y	Correct?

Attempt 2: y = _____

x	y	Correct?

Attempt 3: y = _____

x	y	Correct?

Attempt 4: y = _____

x	y	Correct?

3. Now that you have come up with an equation that relates the x and y values from the correct points table, you will use that equation to determine other points that lie along the same line. In the table below:

a) Chose an x value between -10 and 10 that is not already contained in the correct points table
b) Use your equation to calculate the y value that corresponds to that x value
c) Add that point to the graph on the previous page
d) Check to see if that point lies on the line
e) Repeat a) through d) three more times

If all of they new points lie on the line, then congratulations, you have come up with an equation that accurately describes the line! If not, give it another shot with a new equation.

x	y	On Line?

TEACHING GUIDE

Moving from Points to Lines

Center for Algebraic Thinking

Overview:

In this activity students develop the idea that a line is simply an infinite collection of related points, and are encouraged to develop the mathematical relationship between those points.

Common Core State Standards:

- F-LE.A.2
- F-BF.A.1

Encyclopedia of Algebraic Thinking:

- Analysis of Change: Understanding Slope
- Analysis of Change: Understanding the "b" in "y = mx + b"
- Patterns and Functions: Linear Function

Description:

The *Point Plotter* app begins with nothing more than a line plotted on a coordinate system. The game aspect of the app involves students identifying as many coordinate points as possible that lie on that lie within a specific time limit. Once students can identify at least six correct points, they can begin this activity, which accomplishes two goals. The first goal is to show students that lines are nothing more than a collection of related points. As they get better at the game they will realize that there is no limit to the number of points that they could identify (if given an infinite amount of time). The second goal is for students to see the connection between those points. While this activity provides the space and probing questions to get at that connection, it leaves it for the student to develop and test those connections. That is the strength of this activity - it allows students to come up with their own ideas and then test those ideas by applying them to hard data.

Extensions:

- Moving from Points to Lines: The Follow Up

Moving from Points to Lines:
The Follow Up

For use with the iPad app *Point Plotter*

Center for Algebraic Thinking

Name: Date: Period:

In the first part of this assignment, you developed an equation that related all of the points that lie on a straight line. In this follow up assignment, we will take a closer look at equations that define straight lines - which are called **linear equations**.

Think back to the first part of this assignment... You may have noticed that as you move from one correct point to the next there is a consistent pattern to the changes in the x and y values of those points. In this activity we are going to develop that idea a bit further.

Part 1: Chang in X & Change in Y

The first thing that we are going to do is look at how changes in y and changes in x are related as you move from one point on the line to another. To do that, start by picking two correct points from the table of correct values, and fill in the table below with the x value and y value of those points:

First Point	x_1 =	y_1 =
Second Point	x_2 =	y_2 =

Now we are going to look at how the y values change and how the x values change. Use the four values from above to fill in the table below:

Change in y	$(y_2 - y_1)$ =
Change in x	$(x_2 - x_1)$ =

In the final step, we are going to look at **the ratio** between the change in y and change in x:

Ratio of change in y and change in x	$(y_2 - y_1) / (x_2 - x_1)$ =

That ratio that you found has a special name. It is called the **slope** of the line. Let's see what happens when we use two *different* points to calculate the slope.

You will now complete the same steps for two different correct points. Select two new Correct Points and follow the same steps as before, and fill in the table that follows:

Third Point	x_3 = y_3 =
Fourth Point	x_4 = y_4 =
Change in y	$(y_4 - y_3)$ =
Change in x	$(x_4 - x_3)$ =
Ratio of change in y and change in x	$(y_4 - y_3) / (x_4 - x_3)$ =

If you did everything correctly, then the ratio you end up with when using the third and fourth points should be the same as for the first and second. Is that true for your points?

What this shows is that the **slope of the line** (which, again, is just the ratio you calculated) stays the same, no matter what two points you use to calculate it.

Part 2: The Y-Intercept

Most of the lines that we look at cross the y-axis in just one location (the only exceptions are vertical lines). The y value at that point where the line crosses the y-axis is called the **y-intercept**.

What is the y-intercept for the line that you have been working with in this activity?

Part 3: The Equation

We will now bring the two concepts we have learned about (the **slope** and the **y-intercept**) together to create an equation for the line. The whole goal of writing an equation for the line is to make it easy for us to *calculate* the y value of the line after being given any x value. Since the y value of the line changes as we move left or right along the x-axis (unless the line is horizontal, in which case the line always has the same y value) then that means our equation should also show that same change.

To build our equation for the line, let's start by thinking about what is going on at the point where x is equal to zero. Since that is where the line crosses the y-axis, then the y value of the line just equal to the y-intercept:

y = **y-intercept** (when x equals zero)

Now if we think about moving left or right away from the point where x equals zero, then we can use our **slope** to figure out how much we need to add or subtract to the y-intercept. Recall that slope is just the ratio of the change in y value and the change in x value from one point to another on our line:

slope = (change in y) / (change in x)

We can solve that equation for (change in y) by multiplying both sides by (change in x):

slope * (change in x) = (change in y)

We can now add this change in y to our equation for y that we started on the last page:

y = **y-intercept** + **slope***(change in x)

But since we started the equation when x = 0, our change in x will simply be the x value at the new point:

y = **y-intercept** + **slope***x

So once you know the y-intercept and the slope of the line, you can figure out the y value at any given x value using the equation above.

Our last step is to replace 'y-intercept' and 'slope' with letters so that it is easier to write. By convention we use the letter 'b' for 'y-intercept' and 'm' for 'slope':

y = b + m*x

And finally, by convention we switch the order of the terms on the right side of the equation:

y = m*x + b

That's it! We have finally reached the general equation of a line. The 'general' part means that any line has an equation that can be written in the form you see above. It took some work to build it up, but hopefully you now understand where it came from and see how useful it will be. To make sure it works for the line you've working with in this activity, write the full equation for your line (by inserting values for 'm' and 'b', and then use the equation to calculate a few points to see if they match up with your graph:

y = _____

x	y
-4	
-3	
-2	

x	y
-1	
0	
1	

x	y
2	
3	
4	

TEACHING GUIDE

Teaching Guide Title

Overview:

In this follow up activity to Moving from Points to Lines, students move from a specific line to the general equation for a line and are introduced to the terms 'slope' and 'y-intercept' as they build up the equation themselves.

Common Core State Standards:

- F-LE.A.2
- F-BF.A.1

Encyclopedia of Algebraic Thinking:

- Analysis of Change: Understanding Slope
- Analysis of Change: Understanding the "b" in "y = mx + b"
- Patterns and Functions: Linear Function

Description:

By building on the concrete experience that students developed in the activity Moving from Points to Lines, this activity introduces linear equations in a constructive way that attempts to shed just as much light on the *process of doing mathematics* as the final ideas themselves (in this case, y = mx + b). The activity starts by explaining the concept of slope (using points that students recorded in the first activity), then moves to y-intercept, and finally to the general form of linear equations.

Extensions:

This activity contains quite a bit of information. A useful extension might be for students to condense all of this information while putting it into their own words, maybe in the form of a poster or some other presentation to give to classmates.

TEACHING GUIDE

Function Investigation

Overview:

In this series of activities, students investigate functions by altering the coefficients in front of each term in the expression and then observing the effect that those alterations have on the graph of the function.

Common Core State Standards:

- F-IF.C.7

Encyclopedia of Algebraic Thinking:

- Patterns and Functions: Transforming Functions
- Patterns and Functions: Graphing

Description:

In each of the activities in this series, students are instructed to adjust each coefficient (one at a time) of a particular function, and record their observations of how those adjustments affect the graph of the function. By manipulating the functions in this way, students will build up an understanding of the role of each coefficient (and the function as a whole) by direct observation. When they learn about functions in this way they will be more likely to remember the role of each coefficient. The functions that are included in this series are the following:

- Linear
- Quadratic (Part 1) [$y = ax^2 + bx + c$]
- Quadratic (Part 2) [$y = a(x - h)^2 + k$]
- Cosine
- Sine
- Absolute Value

Extensions:

Additional investigations are included on the last page of each of these activities. Teachers might also want to pair students together so that they can compare their observations and discuss any differences that they might find.

Function Investigation:
Linear

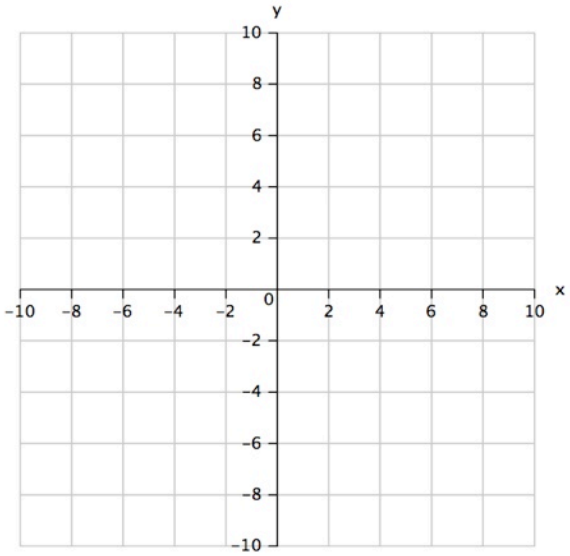
For use with the iPad app *Math Flyer* or *Slope Slider*

Center for Algebraic Thinking

Name: Date: Period:

Select the Premade Graph with the equation **y = mx + b**.

On the graph to the right, draw a sketch of the basic form of this function.



1. What happens when you increase and decrease the variable **m**?

2. Why do you think changes in **m** affect the graph in that way?

3. What happens when you increase and decrease the variable **b**?

4. Why do you think changes in **b** affect the graph in that way?

Additional Investigations

1. What happens when you set **m** equal to zero?

2. What happens when you set **b** equal to zero?

Function Investigation:
Quadratic (Part 1)

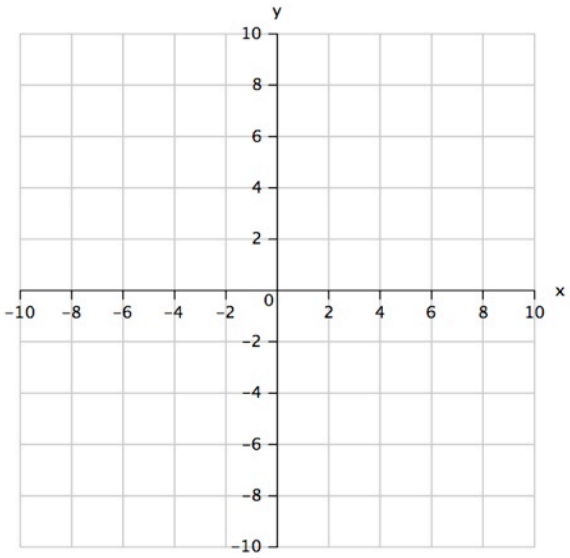
For use with the iPad app *Math Flyer*

Center for Algebraic Thinking

Name: Date: Period:

Select the Premade Graph with the equation $\mathbf{y = ax^2 + bx + c}$.

On the graph to the right, draw a sketch of the basic form of this function.



1. What happens when you increase and decrease the variable **a**?

2. Why do you think changes in **a** affect the graph in that way?

3. What happens when you increase and decrease the variable **b**?

4. Why do you think changes in **b** affect the graph in that way?

5. What happens when you increase and decrease the variable **c**?

6. Why do you think changes in **c** affect the graph in that way?

Additional Investigations

1. What happens when you set **a** equal to zero?

2. What happens when you set **b** equal to zero?

3. What happens when you set **c** equal to zero?

Function Investigation:
Quadratic (Part 2)

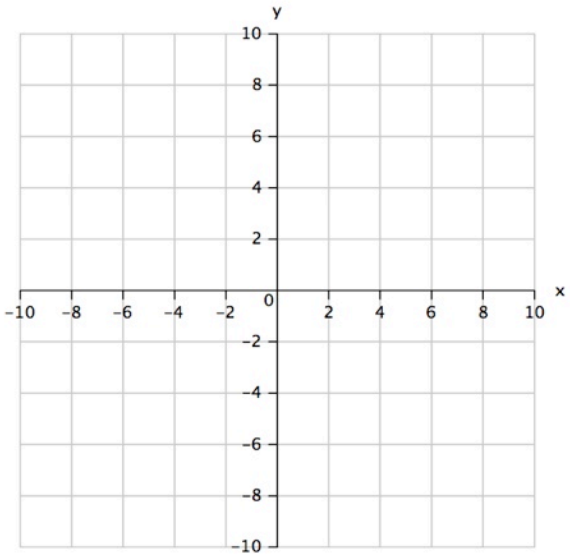
For use with the iPad app *Math Flyer*

Center for Algebraic Thinking

Name: Date: Period:

Select the Premade Graph with the equation $\mathbf{y = a(x - h)^2 + k}$.

On the graph to the right, draw a sketch of the basic form of this function.



1. What happens when you increase and decrease the variable **a**?

2. Why do you think changes in **a** affect the graph in that way?

3. What happens when you increase and decrease the variable **h**?

4. Why do you think changes in **h** affect the graph in that way?

5. What happens when you increase and decrease the variable **k**?

6. Why do you think changes in **k** affect the graph in that way?

Additional Investigations

1. What happens when you set **a** equal to zero?

2. What happens when you set **h** equal to zero?

3. What happens when you set **k** equal to zero?

Function Investigation:
Cosine

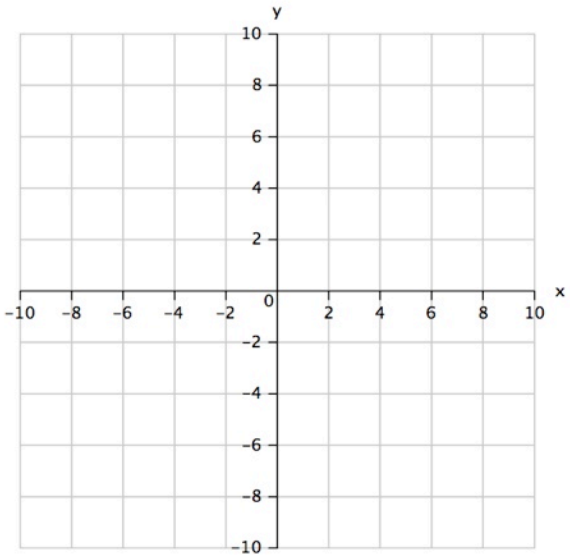
For use with the iPad app *Math Flyer*

Center for Algebraic Thinking

Name: Date: Period:

Select the Premade Graph with the equation **y = a*cos(bx + c) + d**.

On the graph to the right, draw a sketch of the basic form of this function.



1. What happens when you increase and decrease the variable **a**?

2. Why do you think changes in **a** affect the graph in that way?

3. What happens when you increase and decrease the variable **b**?

4. Why do you think changes in **b** affect the graph in that way?

5. What happens when you increase and decrease the variable **c**?

6. Why do you think changes in **c** affect the graph in that way?

5. What happens when you increase and decrease the variable **d**?

6. Why do you think changes in **d** affect the graph in that way?

Additional Investigations

1. What happens when you set **a** equal to zero?

2. What happens when you set **b** equal to zero?

3. What happens when you set **c** equal to zero?

4. What happens when you set **d** equal to zero?

Function Investigation:
Sine

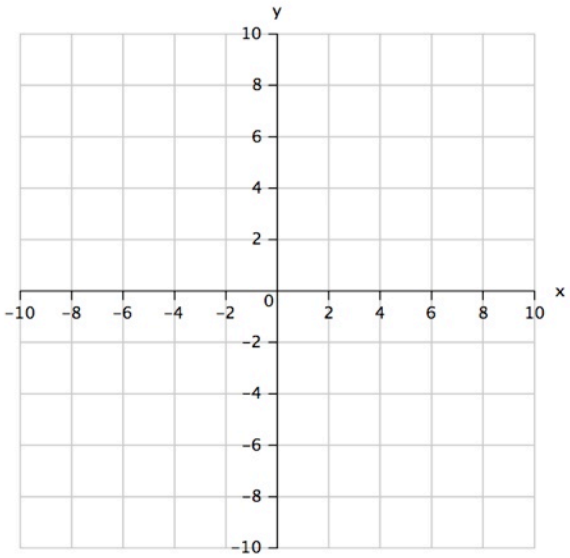
For use with the iPad app *Math Flyer*

Center for Algebraic Thinking

Name: Date: Period:

Select the Premade Graph with the equation **y = a*sin(bx + c) + d**.

On the graph to the right, draw a sketch of the basic form of this function.



1. What happens when you increase and decrease the variable **a**?

2. Why do you think changes in **a** affect the graph in that way?

3. What happens when you increase and decrease the variable **b**?

4. Why do you think changes in **b** affect the graph in that way?

5. What happens when you increase and decrease the variable **c**?

6. Why do you think changes in **c** affect the graph in that way?

5. What happens when you increase and decrease the variable **d**?

6. Why do you think changes in **d** affect the graph in that way?

Additional Investigations

1. What happens when you set **a** equal to zero?

2. What happens when you set **b** equal to zero?

3. What happens when you set **c** equal to zero?

4. What happens when you set **d** equal to zero?

Function Investigation:
Absolute Value

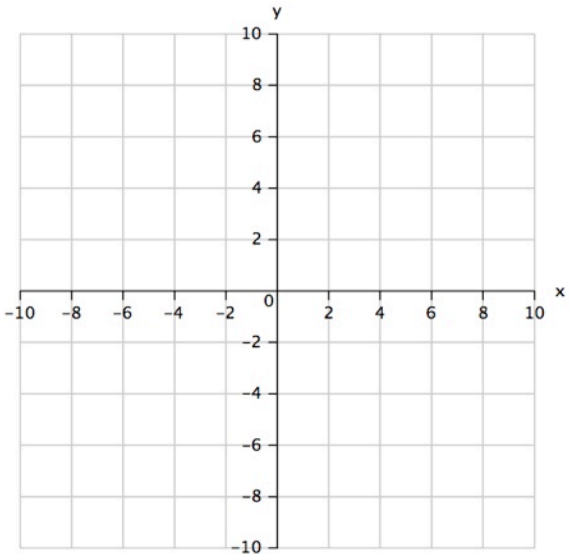
For use with the iPad app *Math Flyer*

Center for Algebraic Thinking

Name: Date: Period:

Select the Premade Graph with the equation **y = a*abs(bx + c) + d**.

On the graph to the right, draw a sketch of the basic form of this function.



1. What happens when you increase and decrease the variable **a**?

2. Why do you think changes in **a** affect the graph in that way?

3. What happens when you increase and decrease the variable **b**?

4. Why do you think changes in **b** affect the graph in that way?

5. What happens when you increase and decrease the variable **c**?

6. Why do you think changes in **c** affect the graph in that way?

5. What happens when you increase and decrease the variable **d**?

6. Why do you think changes in **d** affect the graph in that way?

Additional Investigations

1. What happens when you set **a** equal to zero?

2. What happens when you set **b** equal to zero?

3. What happens when you set **c** equal to zero?

4. What happens when you set **d** equal to zero?

Graphing the Terms of Polynomials

For use with the iPad app *Math Flyer*

Center for Algebraic Thinking

Name: Date: Period:

1. Use Math Flyer to graph the following four equations, all on the same screen:

a.) $y = 3$ b) $y = 2x$ c) $y = x^2$ d) $y = x^2 + 2x + 3$

2. Use the data table within the app or your own computing skills to complete the following table:

x	y = 3	y = 2x	$y = x^2$	$y = x^2 + 2x + 3$
-3	3	-6	9	6
-2				
-1				
0				
1				
2				
3				

3. Now look at the graph of the four equations. What connection do you see between the graph of equation d) and the graph of the other three equations?

TEACHING GUIDE

Graphing the Terms of Polynomials

Overview:

in this activity, students graph all terms of a polynomial individually, as well as the complete polynomial, in order to see how each term contributes to the function as a whole.

Common Core State Standards:

- F-IF.C.7

Encyclopedia of Algebraic Thinking:

- Patterns and Functions: Transforming Functions
- Patterns and Functions: Graphing

Description:

The *Math Flyer* app makes it easy to graph multiple functions on the same screen. In this activity, students use that ability to build up an understanding of how each term in a polynomial contributes to the graph of that polynomial. After graphing each of the terms of a polynomial, as well as the complete polynomials, students then complete a table of values for each of the terms to reinforce the idea that the picture of the complete polynomial is simply the addition of all of the terms of that polynomial. By seeing this in two different representations (graph and table), students will be more likely to identify this relationship on their own and more likely to remember it in the future.

Extensions:

Teachers can extend this activity by giving students additional polynomials and asking them to analyze the new polynomials in the same way that they analyzed the polynomial in this activity.

Submariner Algebra Workspace

For use with the iPad app *Submariner Algebra*

Name: Date: Period:

Use the space below to show the work needed to calculate the slope and y-intercept of the enemy's submarine route after you have gotten two hits.

Round One:

Hit #1: (____ , ____) Hit #2: (____ , ____)

Workspace:

Slope: Y-Intercept:

Round Two:

Hit #1: (____ , ____) Hit #2: (____ , ____)

Workspace:

Slope: Y-Intercept:

Round Three:

Hit #1: (____ , ____) Hit #2: (____ , ____)

Workspace:

Slope: Y-Intercept:

Round Four:

Hit #1: (____ , ____) Hit #2: (____ , ____)

Workspace:

Slope: Y-Intercept:

Round Five:

Hit #1: (____ , ____) Hit #2: (____ , ____)

Workspace:

Slope: Y-Intercept:

TEACHING GUIDE

Submariner Algebra Workspace

Overview:

This activity provides a space for students to show their work as they play the game *Submariner Algebra*, in which they must determine the line that an enemy submarine follows after locating two points that the submarine crosses (with a gameplay that resembles the board game Battleship).

Common Core State Standards:

- F-LE.A.2
- F-BF.A.1

Encyclopedia of Algebraic Thinking:

- Coming Soon

Description:

The app *Submariner Algebra* does not require a paper resource to accompany it, but this resource serves two functions. The first is to encourage students to work through the calculation of slope and y-intercept by hand, rather than trying to do it in their head based on the points that they have identified as hits. This reinforces the good habit of showing your work. The second function is to provide accountability - the teacher can collect this record of students' work at the end of the activity to see both the quality of the students' work and the quantity - how many rounds students were able to complete during the allotted time for this activity.

Extensions:

If you have multiple iPads in the classroom, students can use this resource in the 2-Player Mode to challenge each other. You could even set up a tournament so that each student plays multiple rounds against various opponents.

Submariner Algebra
The Paper Version

For use with the iPad app *Submariner Algebra*

Name: Date: Period:

In the iPad app Submariner Algebra *you battle your opponent to see who can determine the path of the opponent's submarine first. In this activity you will bring the battle into the real world.*

Instructions

Setting Up

(1) Make sure that your opponent cannot see your playing board (on the next page) and that you cannot see theirs.

(2) Select the path of your submarine, and write the equation for that path on the following page.

(3) Draw the path of your submarine on the coordinate system.

(4) Determine the marks that you will use for you opponent's guesses, your incorrect guesses, and your correct guesses. Add each mark to the **KEY** for reference.

Gameplay

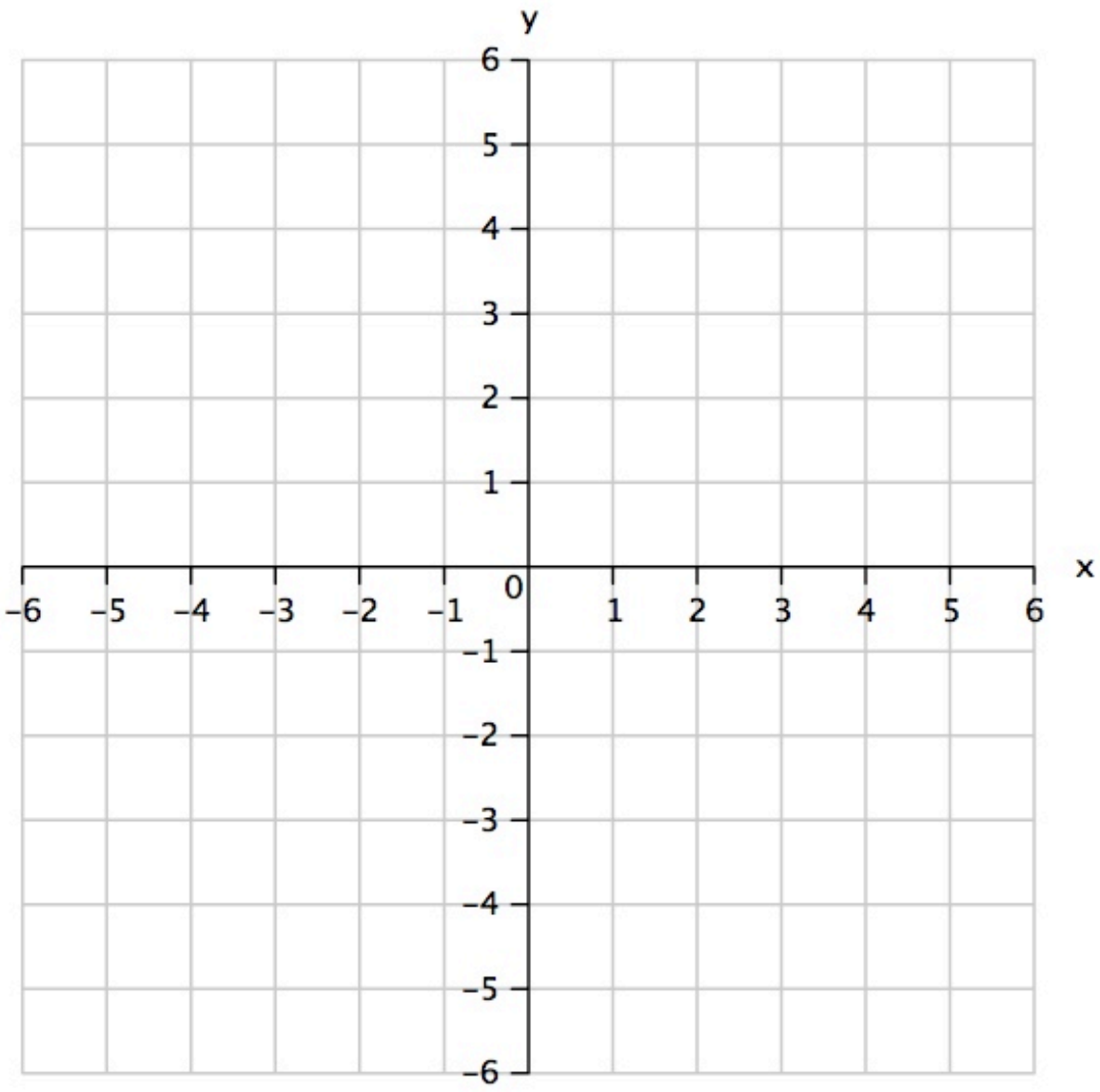
(1) You and your opponent alternate guesses, adding the appropriate marks to your coordinate system for each guess - a hit or a miss.

(2) Once you have two hits, determine the path of your opponent's submarine, and state the equation of the path to your opponent to see if you are correct or not.

(3) First team to correctly guess the path of their opponent wins!

Your Submarine's Path: y = _____

KEY Opponents Guesses: ____ Your Incorrect Guesses: ____ Your Correct Guesses: ____



Workspace:

Your Opponent's Path: y = _____

TEACHING GUIDE

Submariner Algebra - The Paper Version

Overview:

In this activity students follow the same gameplay as that found in the iPad app *Submariner Algebra*, but they play the game on paper instead of an iPad.

Common Core State Standards:

- F-LE.A.2
- F-BF.A.1

Encyclopedia of Algebraic Thinking:

- Coming Soon

Description:

There are a variety of reasons that you might want use the paper version of this game. One is that you might not have enough iPads for all students. With this paper version you could have the entire class playing at once, some with iPads, others with pencil and paper. Another reason is because of the added responsibilities on the student when playing on paper. Rather than just tapping a location, students will have to list a coordinate point, and respond to the guesses of their opponent. They will also have to evaluate the correctness of the path that their opponent guesses, maybe recognizing that they determined the y-intercept correctly but didn't get the slope right. Even the simple act of letting students chose their own marks for hits and misses could lead to increased student engagement during the activity. For these reasons it might be worth giving this activity a try in conjunction with the iPad version.

Extensions:

After a few rounds of play you could have students complete the Developing Game Strategies activity, and then discuss their approaches with each other.

Tortoise and the Hare Algebra Challenge Progress Monitor

For use with the iPad app *Tortoise and the Hare Algebra*

Center for Algebraic Thinking

Name: Date: Period:

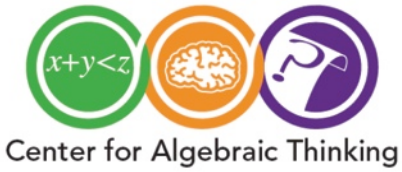
Use this sheet to keep track of your progress as you complete the in-app challenges.

	Solution	Check
Level: Challenge:		
Level: Challenge:		
Level: Challenge:		
Level: Challenge:		

	Solution	Check
Level: Challenge:		
Level: Challenge:		
Level: Challenge:		
Level: Challenge:		
Level: Challenge:		
Level: Challenge:		

TEACHING GUIDE

Tortoise and the Hare Algebra Progress Monitor



Overview:

This resource is a way for students to keep track of their progress in the 'Challenge' section of the iPad app *Tortoise and the Hare Algebra*.

Common Core State Standards:

- A-CED.A.1
- A-CED.A.2
- F-LE.A.2
- Modeling

Encyclopedia of Algebraic Thinking:

- (waiting for permanent website to be up and running)

Description:

The challenges provided in this app ask a variety of questions about particular scenarios that would play out as the tortoise and the hare race at different speeds. The beauty of the app is that no key is needed - students can simply input the conditions, tap 'Start Race,' and then watch to see if the results confirm that they answered the challenge correctly. This paper resource allows students to keep track of which challenges they have completed successfully as well as record their solutions and the process of arriving at that solution.

Extensions:

Teachers could extend this activity by having students *create their own challenges* that either they themselves or a classmate has to solve.

Activity Title

Activity Subhead (if needed)

For use with the iPad app *App Name*

Center for Algebraic Thinking

Name: Date: Period:

TEACHING GUIDE

Teaching Guide Title

Overview:

Overview text

Common Core State Standards:

- Standard 1
- Standard 2

Encyclopedia of Algebraic Thinking:

- Entry 1
- Entry 2

Description:

Description text

Extensions:

Extensions text